An Algorithm for Locating Point Scatterers Using a Wide-Band Wireless Network

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Abstract—A map of the coordinates of the most significant point scatterers in the environment surrounding a wireless network would be useful in many contexts, e.g., in indoor positioning and mapping applications, for channel estimation, and also for wake-up radio schemes. In this work, we investigate the feasibility of creating such a map based on signal propagation path length data gathered at a number of wide-band network transceivers. Differently from previous approaches to this problem, we propose to formulate the point scatterer mapping problem as an assignment problem. Computer simulations show that, under a few simplifying assumptions, it is indeed feasible to position point scatterers using the proposed algorithm.

I. INTRODUCTION

Recently, Pedersen and Fleury showed how realistic channel models can be obtained using so-called stochastic propagation graphs [1]. A propagation graph is a directed graph where the coordinates of the main scatterers in the environment make up the vertices, and graph edges symbolize wave propagation between scatterers. Given a realization of such a graph, a translation into a traditional channel impulse response between two transceivers is possible. The main difference between this scheme, and most previous approaches to channel modelling, is that here the environment itself is modelled, as opposed to just modelling the response of the environment.

In this work, we investigate the feasibility of solving the inverse problem to that considered by Pedersen and Fleury, i.e., to find a propagation graph that agrees with multipath observations made at a number of wide-band network transceivers. A “map” of the coordinates of the most significant point scatterers in the network deployment area could be useful in many contexts, e.g., as a-priori information in channel estimation algorithms, as input to indoor mapping algorithms, in positioning applications, see, e.g., [2], and also for so-called wake-up radio systems described, e.g., in [3].

Similar problems have been studied before, see, e.g., [4], and references cited therein. The solution proposed in [4], and also elsewhere in the literature, is based on an exhaustive search for scatterers over a pre-defined grid that covers the area of interest. In short, it is assumed that each transceiver pair is able to measure the propagation path lengths of the most significant specular multipath components on the link between them. An objective-function based on the measured path lengths and a-priori known transceiver positions is then evaluated at each grid vertex. A high vertex objective function value is taken as an indication that a reflective object is present at that position.

There are three principal problems associated with this approach; (a) The grid will, in general, contain a large number of “false” objective function peaks, i.e., some grid vertices may have a high objective function value even though they are not close to the position of a scatterer. It is therefore difficult to determine the number of significant scatterers, i.e., the number of vertices in the underlying propagation graph. (b) Even if only a single scatterer position estimate is sought, the probability of selecting a false peak is often high. (c) The complexity of this approach is tightly coupled to the grid resolution and span. The coverage area and grid resolution is therefore, in practice, limited by the allowable system complexity, and trade-offs between coverage and resolution may be necessary.

In this work, we take an entirely different approach to the scatterer mapping problem. Similar to [4], we assume that the communication system bandwidth is such that a majority of point scatterers generate signal components that can be resolved at the receiving nodes. We also assume that the arrival times of these signal components can be measured and used to compute estimates of propagation path lengths. Arrival-time measurements can be obtained, e.g., via a cross-correlation between the received signal and a known template at both nodes [5], and path lengths can be computed using knowledge of the electromagnetic propagation speed. The details of the path length estimation procedure is not considered more closely here, but work on this problem is ongoing. Differently from the work in [4], we do not assume that all transceiver pairs are able to detect the multipath components from all scatterers. Under these
assumptions, the positioning problem can be formulated as an assignment problem. However, the complexity of the resulting problem does not permit a brute-force attack. Hence, we approach the problem using Lagrangian duality, coupled with a hypothesis pruning heuristic. The proposed algorithm is similar to that proposed in [6], which was originally intended for radar target tracking applications.

The remainder of this work is organized as follows. In Sec. II, we formalize the positioning problem. Our approach to solving the problem is presented in Sec. III. We evaluate the performance of the proposed algorithm in Sec. IV, and the paper is concluded in Sec. V.

II. PROBLEM FORMULATION

A. Signal model

Suppose $N$ wireless transceivers are randomly located in a given region, and their positions in two or three dimensions are (approximately) known. The transceivers form $J \leq N(N-1)/2$ pairs. A pair is formed if two transceivers together can estimate the lengths of the most significant propagation-paths between them. Two-way connectivity is required for two nodes to form a pair, but we do not require the nodes to be synchronized [5]. The propagation path length, shown in Fig. 1, for pair $j$ and a point scatterer at coordinates $s_i$ in two or three dimensions, is

$$d_j(s) = \|x_{j,1} - s\| + \|x_{j,2} - s\|, \quad (1)$$

where $\|\cdot\|$ denotes Euclidean norm, and $x_{j,1}$, $x_{j,2}$ are the a-priori known coordinates of the transceiver nodes of pair $j$. An estimate $\hat{d}_j = d_j(s) + n$ is available at pair $j$ if both nodes detect the signal from the other node over the scatterer at $s$. Here, $n$ is a random error term with a distribution that depends on the communication system and ranging algorithm. Distance estimates at a given pair may also correspond to the line-of-sight (LOS) path, or paths over several point scatterers, as indicated in Fig. 1. In these cases (1) may impose a significant modeling error. We make no assumptions on a-priori knowledge of when such events occur. Distance estimates that do not fulfill the triangle inequality, i.e., when $\hat{d}_j < \|x_{j,1} - x_{j,2}\|$, are removed from consideration.

B. Least-squares position estimation

Suppose we want to estimate the coordinates $s_l \in \mathbb{R}^d$ of the $l$th scatterer. If pair $j$ detects $D_j$ significant propagation paths, and computes distance estimates $\hat{d}_{j,1}, \ldots, \hat{d}_{j,D_j}$, we have a total of $D = \sum_{j=1}^J D_j$ distance estimates. These estimates must first be assigned to the different scatterers. Let $a_l \in \mathbb{Z}^J$ be a vector with elements $[a_l]_j \in \{0, \ldots, D_j\}$, and $d(a_l) \in \mathbb{R}_+^J$ be the vector of estimates, $[d(a_l)]_j = \hat{d}_{j,[a_l]_j}$, assigned to the $l$th scatterer. We take $[a_l]_j = 0$ to mean that the $l$th scatterer was not detected by one or both nodes of pair $j$. For notational convenience, and with a slight abuse of notation, we let $d_{j,0} = d_j(s) + \sigma_0$, where $\sigma_0 \geq 0$ is a fixed penalty parameter discussed below.

Given an assignment vector $a_l$, the weighted least-squares (WLS) estimate of the scatterer coordinates $s_l \in \mathbb{R}^d$ is

$$\hat{s}_l = \arg \min_{s \in \mathbb{R}^d} \left\|d(a_l) - d(s)\right\|_V^2, \quad (2)$$

where $d(s) \in \mathbb{R}_+^J$ is a vector with elements $[d(s)]_j = d_{j,0}$, and $V$ is a symmetric positive definite weighting matrix which, in general, depends on $a_l$. The contribution to the LS residual in (2) due to a dummy estimate $d_{j,0}$ is $\sigma_0^2$. If $\sigma_0$ is set large, then assignments with few hypothesized detections are penalized, and vice versa. Of course, (2) only makes sense if at least $d$ distance estimates are available.

A practical implementation of (2) using, e.g., gradient-descent based search methods may converge to local minima. To alleviate this problem, we use the POCS algorithm described in, e.g., [7], together with a gradient-descent based search implemented by the lsqnonlin routine in MATLAB R2007a [8]. In POCS, a point in the plane (or space) is iteratively projected onto convex sets, that are defined by the distance estimates (in this case elliptic disks), in order to find a point in their intersection. This point is then used to initialize the gradient-descent search. To reduce complexity, we inscribe ellipses in rectangles (or ellipsoids in cuboids), and use the rectangles instead of ellipses as input in the POCS algorithm.

C. Data association problem

Let $A = \{a \in \mathbb{Z}^J : 0 \leq [a]_j \leq D_j, \ j = 1, \ldots, J\}$ be the set of all possible assignments. We first note that many vectors in $A$ constitute unlikely assignments that can be removed from consideration before attempting to assign path length estimates to scatterers, without significant loss in performance. A pruned set $A' \subseteq A$ is formed as follows. For $a \in A'$, then: (a) $d(a)$ has at least $d$ non-dummy estimates, (b) after inscribing ellipses defined by $d(a)$ and transceiver positions (Fig. 1) in rectangles, all such rectangles intersect at least one of the other rectangles, (c) no such rectangle is completely contained by another rectangle.
Due to the rectangular approximation of ellipses, tests (b) and (c) can be efficiently implemented on a computer. Examples of where the last two criteria does not hold is given in Fig. 2. Numerical simulations show that these simple pruning criteria effectively reduces the cardinality of $A_l$, and therefore also the complexity of the assignment problem.

![Assignment fails by criteria (b) and (c)](image)

**Fig. 2.** Example of pruning criteria (b), left, and (c), right

Under the assumption that a point scatterer generates a single specular signal component, each propagation path length estimate is to be associated with at most one scatterer. Thus, the positioning problem for $L$ scatterers can be formulated as

$$
\min_{\{a_l\}_{l=1}^L} f = \sum_{l=1}^L \sum_{a_l \in A} \left( \min_{d(s) \in \mathbb{R}^2} \left\| d(\hat{a}) - d(s) \right\|^2_{V(\hat{a})} \right) I_{a_l},
$$

such that:

(A) $\sum_{a_l \in A} I_{a_l} = 1$, $l = 1, 2, \ldots, L$, and,

(B) $\sum_{l=1}^L \sum_{a_l \in B_j, n_j} I_{a_l} \leq 1$, $j = 1, \ldots, J$, $n_j = 1, \ldots, D_j$,

where $B_{j,n_j} = \{ a \in A : [a]_j = n_j \}$, and $I_{a,b}$ is an indicator function such that $I_{a,b} = 1$ when $a = b$ and zero otherwise. The $L$ constraints in (A) ensure that $a_l \in A$, while the $\sum_{j=1}^J D_j$ constraints in (B) ensure that each distance estimate is associated with at most one point scatterer.

We emphasize that the algorithm proposed below does not break down if all $L$ scatterers are not resolved by the path length estimator, neither does it brake down if all available path length estimates have been made on a LOS path or on paths over more than one scatterer.

**III. A SOLUTION TO THE ASSIGNMENT PROBLEM BASED ON LAGRANGIAN RELAXATION**

A brute-force approach to solving the problem in (3), even after pruning $A$ down to $A'$, is not very efficient, and may not even be computationally feasible. We therefore approach the problem using Lagrangian relaxation techniques. We only give a very brief overview of the algorithm here, and refer to the work in [6], [9], [10], and references cited therein, for a more detailed exposition.

We assume we have an estimate $\hat{L}$ of $L$ (if not, we initialize at $L = 1$ and iteratively adjust this estimate as described in Sec. III-D). The proposed algorithm is an iterative algorithm where each iteration involves three main steps, as outlined in sections III-A, III-B, and III-C below.

**A. Relaxation**

Relax all constraints in (3) except those in (A) and those associated with estimates from one arbitrary pair, here chosen to be the first pair ($j = 1$). Denote the dual variable vector by $\mu$, and the element in $\mu$ associated with distance estimate $d_{j,k}$ by $\mu_{j,k}$. The dual function $q(\mu)$ is then

$$
q(\mu) = \inf_{\{a_l\}_{l=1}^L} \left\{ \sum_{j=2}^J \sum_{n_j=1}^{D_j} \mu_{j,n_j} \left( \sum_{l=1}^L \sum_{a_l \in B_j, n_j} I_{a_l} - 1 \right) \right. \\
+ \sum_{l=1}^L \sum_{a_l \in A} \left( \min_{s \in \mathbb{R}^2} \left\| d(\hat{a}) - d(s) \right\|^2_{V(\hat{a})} \right) I_{a_l} \left. \right\}
$$

such that: (A) $\sum_{a_l \in A} I_{a_l} = 1$, $l = 1, 2, \ldots, L$, (B) $\sum_{l=1}^L \sum_{a_l \in B_j, n_j} I_{a_l} \leq 1$, $n_j = 1, 2, \ldots, D_j$, (C) $\mu_{j,n_j} \geq 0$, $j = 2, 3, \ldots, J$, $n_j = 1, 2, \ldots, D_j$.

For a given vector $\mu$, the assignment problem in (4) is two-dimensional, and can be solved in polynomial time with the auction algorithm, due to Bertsekas [10].

**B. Transforming into a primal feasible assignment**

As in [6], relaxed constraints from (B) in (3) are now enforced pair by pair, again using the auction algorithm. The utilities in the auction algorithm for assigning the distance estimates of pair $j$ are the best achievable least-squares residuals given a fixed assignment up to pair $j-1$ and modified by the dual variables. This will lead to a mapping between scatterers and estimates that is feasible in (3).

**C. Algorithm termination criteria and dual variable update**

The assignment found after enforcing constraints (Sec. III-B) is, at least during the first iterations, likely to be sub-optimal in (3). However, the weak duality theorem [6] states that, for any vector $\mu \geq 0$, we have $q(\mu) \leq f$. Hence, for each feasible assignment found above (Sec. III-B), the difference between its objective function value $f$ in (3) and its infimum $f^*$ is easily bounded. As a measure of merit at iteration $i$, we use the relative duality gap, which is given by

$$
\Delta = \frac{\min_{\nu \in \{1, \ldots, i\}} f^{(\nu)} - \max_{\nu \in \{1, \ldots, i\}} q(\mu^{(\nu)})}{\min_{\nu \in \{1, \ldots, i\}} f^{(\nu)}},
$$
where $\nu$ is an iteration index. The algorithm is terminated when $\Delta$ goes below a pre-specified threshold.

If the best found solution at iteration $i$ is not satisfactory (as indicated by $\Delta$), we update dual variables and iterate again. The dual variable update is the same as that used in [6], which is similar to the sub-gradient method, see, e.g., [10] for details. Intuitively, this update scheme can be interpreted as follows: If, after fixing the assignment up to and including pair $p$, two or more scatterers have a given distance estimate as their preferred choice in terms of least-square residual, then the “price” (or dual variable) of this estimate is increased. In the next iteration of the algorithm, the LS residuals of all assignments are increased by the sum of prices of distance estimates in that assignment.

### D. Finding $L$

Finally, after the algorithm has converged or been otherwise terminated, we investigate the LS residuals for the $L$ positioned scatterers. If the residuals of some scatterer position estimates are high, we reduce $L$ and run the algorithm again. On the other hand, if many unassigned strong propagation path length estimates exist, we increase $L$. We note that the choice of method for updating $L$ is application-specific and could involve a-priori information. Such methods are, however, not considered more closely here.

### IV. Numerical Results

To make the simulated scenarios system independent, we evaluate results in terms of distance units (u), i.e., not meters. One squared distance unit corresponds to the area of the network deployment region. We assume all path length estimates have the same error variance, and therefore the weighting matrix $V(a)$ is taken to be the identity matrix $I$, $\forall a \in A$.

As a comparison to the proposed algorithm, the exhaustive grid-search based algorithm proposed by Chang and Sahai in [4] was implemented. In this algorithm, a “score” is computed for each vertex in a grid. We assume this algorithm has a-priori information about the network deployment area, and thus a suitable grid span is known. The score at grid point $x \in \mathbb{R}^d$ for $J$ transceiver pairs is defined in [4] as

$$\text{score}(x) = \sum_{j=1}^{J} \max \left\{1 - \min_{n_j \in \{1, \ldots, D_j\}} K \| \hat{d}_{j,n_j} - d_j(x) \|, 0 \right\},$$

where $d_j(x)$ and $\hat{d}_{j,n_j}$ was defined in Sec. II, and $K$ is a tuning parameter that was set to unity. As noted in Sec. I, the computational complexity of this algorithm is tightly coupled to the resolution of the grid, and the covered area (the span of the grid). In Fig. 3, an example of grid score in a network with $N = 5$ nodes, $J = 5$ pairs, and $L = 4$ point scatterers is plotted. No noise or other perturbations affects path length estimates, and the grid resolution is 0.05 u. Since three of the four highest score points are clustered together, we cannot simply take the four grid coordinates with highest score as the position estimates. Thus, for $L > 1$, additional processing of the score matrix is needed to produce position estimates. The proposed algorithm, on the other hand, was in this case able to correctly locate all four point scatterers.

#### A. Impact of a Gaussian noise

A network with $N = 5$ nodes and $L = 3$ scatterers was randomly deployed over a square area with side 1 u. We randomly selected $J = 5$ pairs out of the ten possible pairs as connected, and the rest were assumed unconnected. Here (unknown to the algorithm) we let all pairs detect the three scatterers. Two spurious path length estimates, uniformly distributed in $[0, 1]$, was added to each pair. A penalty parameter of $\sigma_0 = 0.05$ u was used. The algorithm was terminated after 20 iterations, or when a relative duality gap $\Delta \leq 10^{-3}$ was reached. We let propagation path length estimates be affected by an additive white Gaussian zero-mean random variable with standard deviation $\sigma_u u$. This type of error distribution can be expected after averaging distance estimates over time, especially if the estimates are affected by uniformly distributed errors due to random correlator sampling, see, e.g., [5].

The algorithm was run for 1000 random network layouts and noise realizations. The performance of the algorithm is evaluated as follows. After each simulation run, we compute a distance matrix between all relay position estimates and true relay positions. We then select the smallest error, remove the corresponding estimate and relay position from the matrix, and repeat the process until all estimates have been assigned an error. This ensures that no two errors are computed with respect to the same relay. The cumulative density function (CDF) of the positioning error is shown in Fig. 4 for two cases.

1) Case one: Here $L = 1$, allowing us to make a fair comparison with the grid-based approach described above. The error CDF curves are plotted for $\sigma_u = 0.01$ u and $\sigma_u = 0.02$ u and for grid resolutions $G_r = 0.01$ u
and $G_r = 0.05 \text{ u}$. It was noted that the proposed algorithm occasionally produces outliers, i.e., abnormally large errors, caused by convergence to local minima, while the grid-based approach show outliers due to erroneous peak selection.

2) Case two: Here we attempt to locate all three scatterers using the proposed algorithm, we have not included a comparison with the grid-based approach due to the problems in associating score peaks to scatterers mentioned above. Note that the percentage of outliers increases with noise variance, and that the performance for $L = 3$ is only slightly worse than that for $L = 1$.

To put the results of Fig. 4 into context, we note that $\sigma_d = 0.01 \text{ u}$ is close to the performance expected from an ultra-wide band (UWB) system deployed over a square area with side 10 m, at high SNR and with a bandwidth of around 500 MHz, such as the system described in [5]. Simulations (not shown here) were also run with a zero-mean circular Gaussian positioning error with variance $\sigma_p^2$ affecting the a-priori known transceiver positions. With $\sigma_p^2 = 0$ and $\sigma_0 = 0.05 \text{ u}$, the positioning error for $\sigma_p = 0.02 \text{ u}$ was below 0.1 u in approximately 90 percent of simulation runs, and below 0.05 u in approximately 80 percent of cases.

V. CONCLUSIONS AND FUTURE WORK

We have investigated the feasibility of locating a number of strong point scatterers in the deployment area of a wireless network. Differently from previous related work, we have formulated this problem as an assignment problem, and proposed and algorithm to solve it. The algorithm showed promising results in terms of positioning error and robustness to inaccurate a-priori data. The main drawback of the proposed algorithm is convergence to local minima in the LS objective function. The effect of such local minima would be less severe if accurate a-priori initialization points were available in the LS implementation.

We note that the proposed algorithm is not specific to the scatterer positioning problem, but is applicable also to the problem of positioning $L$ relays in the environment surrounding the wireless network.

On-going work currently aims at developing a robust algorithm for accurate propagation path length estimation, and incorporating such physical layer simulations into the overall system simulations. We are also investigating the possibilities of approximating scatterer characteristics in terms of, e.g., volume.

REFERENCES


