Fiber-optic communications with microresonator frequency combs

ATTILA FÜLÖP

Department of Microtechnology and Nanoscience (MC2)
CHALMERS UNIVERSITY OF TECHNOLOGY
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Front cover illustration: Sketch showing the lines from a microresonator comb being modulated with data encoded using QPSK, 16QAM and 64QAM.

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Abstract

Modern data communication links target ever-higher information throughput. To utilize the available bandwidth in a single strand of fiber, optical communication links often require a large number of lasers, each operating at a different wavelength. A microresonator frequency comb is a chip-scale multi-wavelength laser source whose spectrum consists of multiple evenly spaced lines. As the line spacing of a microresonator comb is on the order of several tens of GHz, it provides a promising light source candidate for implementing an integrated multi-wavelength transceiver. The interest for using microresonator combs in communications applications has therefore increased greatly in the last five years. The application-related developments have been complemented with an increased exploration and understanding of the operating principles behind these devices.

This thesis studies microresonator frequency combs in both long-haul and high data-rate (multi-terabit per second) fiber communications systems. The results specifically include the longest demonstrated communications link with a microresonator light source as well as the highest order modulation format demonstration using any integrated comb source. The used microresonators are based on a high-Q silicon nitride platform provided by our collaborators at Purdue University. Part of the results are enabled by the high line powers resulting from a recently demonstrated novel comb state. This state bears similarities with dark solitons in fibers in that it corresponds to a train of dark pulses circulating inside the microresonator cavity. Overall, the results in this thesis provide a promising pathway towards enabling a future chip-scale multi-wavelength coherent transmitter.

Keywords: Fiber-optic communication, integrated optics devices, nonlinear optics, four-wave mixing, microresonators.
Sammanfattning


Den här avhandlingen beskriver och utvärderar mikroresonatorer för kommunikationstillämpningar för både lång-distanslänkar och länkar med hög datatakter (dvs. flera terabit per sekund). De inkluderade resultaten beskriver den längsta demonstrerade länken där mikroresonatorer använts samt en demonstration med den hittills högsta ordningens modulationsformat som använts med någon intregrerad kamkälla. Våra mikroresonatorer är baserade på en hög-Q kiselnitridplattform som tillverkats av våra samarbetspartners på Purdue University i USA. Delar av resultaten har möjliggjorts av de höga linjeeffekterna som en nyligen upptäckt kamtyp genererade. Denna kamtyp liknar mörka solitoner i klassiska fibersystem i att den motsvaras av en cirkulerande mörk puls i resonatorkaviteten. Sammanfattningsvis pekar resultaten i avhandlingen ut en lovande riktning för att möjliggöra framtida intregrerade flerkanaliga koherenta optiska sändare.
Kivonat


Ez a dolgozat a mikrorezonatórok nagytávolságú és magas adatmennyisége (több mint egy pár terabit per másodperc) kommunikációs alkalmazásáról szól. A bemutatott eredmények között említendő az eddigi leghosszabb távolságon működő mikrorezonátor-alapú kommunikációs rendszer. Ezenkívül, a csatolt cikkekben bemutatjuk azt a kísérleti rendszert, amelyben integrált frekvenciafősűkre eddig elért legmagasabb rendű modulációt megvalósítottuk. A mikrorezonátorainkat a Purdue Universityn dolgozó partnereik állították elő magas-Q szilícium-nitrid alapú platformon. Az eredmények egy részét egy újonnan kifejlesztett fősű típusú magas energiájú vonalai tettek lehetővé. Ez a típusú fősű a klasszikus üvegszálakban terjedő sötét szolitóhoz hasonlóan, a rezonátorban keringő sötét pulzusoknak felel meg. Végül, a dolgozatban bemutatott eredmények a jövőbeli integrált többszálú koherens optikai adóvevők fejlesztésének egy igéretes irányát adják meg.
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First of all, I would like to thank my supervisor, Associate Prof. Victor Torres-Company, without his continuing active support and knowledge I would not have made it this far. I would additionally also like to thank my examiner Prof. Peter Andrekson for discussions and lab support and Prof. Magnus Karlsson for discussions and teaching of the relevant topics.

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Last, but not least, I would like to thank all of my family and friends for being supportive and great to be around.
List of Papers

This thesis is based on the following appended papers:


Related publications and conference contributions by the author, not included in the thesis:


Abbreviations

AWGN additive white Gaussian noise.
BER bit error ratio.
CMOS complimentary metal-oxide-semiconductor.
CW continuous-wave.
DSP digital signal processing.
EDFA erbium-doped fiber amplifier.
FEC forward error correction.
FSR free spectral range.
FWHM full width at half maximum.
FWM four-wave mixing.
LO local oscillator.
MI modulation instability.
MZM Mach-Zehnder modulator.
NLSE nonlinear Schrödinger equation.
OOK on-off keying.
OSNR optical signal-to-noise ratio.
QAM quadrature amplitude modulation.
RF radio frequency.
SNR signal-to-noise ratio.
WDM wavelength-division multiplexing.
Contents

Abstract i
Sammanfattning ii
Kivonat iii
Acknowledgements v
List of Papers vii
Abbreviations xi

1 Introduction 1
  1.1 This thesis ................................................. 3

2 Optical frequency combs in coherent communications 5
  2.1 Coherent communications systems ........................ 5
  2.2 Wavelength-division multiplexing ........................ 7
  2.3 Performance metrics and requirements ........................ 8
    2.3.1 Modulation-format dependent requirements .............. 9
    2.3.2 Phase noise ............................................. 12
  2.4 Optical frequency comb technologies ........................ 13
  2.5 Combs as WDM light sources .............................. 16

3 Microresonator dynamics 19
  3.1 Microresonators in the linear regime ..................... 19
    3.1.1 Inter-waveguide coupling ............................. 21
    3.1.2 System characterization ............................... 21
    3.1.3 Chromatic dispersion ................................ 23
3.1.4 Quality factors .................................. 24
3.1.5 Optical confinement ............................. 25
3.1.6 Critical coupling ............................... 25
3.2 Nonlinear propagation ............................... 26
3.3 The Ikeda map ................................ 27
3.4 The Lugiato-Lefever equation .................. 28
3.5 Bi-stability and comb initialization ............ 29
3.6 Thermal effects and detuning .................. 33

4 Basic soliton dynamics in microresonators 35
   4.1 Stable comb states .......................... 35
   4.2 Bright solitons ............................... 36
   4.3 Dark pulses ................................. 40

5 Future outlook 43

6 Summary of papers 45

References 48

Appendices 69
   Offline coherent receiver DSP .................... 69
   Microresonator simulation ...................... 71
   Microresonator stage ........................... 72

Papers 77
Chapter 1

Introduction

The need to communicate and send messages between people has existed since the invention of the written word. For most of the history of humankind this has involved sending messages by carrier, either humans or carrier animals. While there are modern suggestions for carrying internet protocol packets using avian carriers [1], the vast majority of data traffic has been transmitted on lower-latency links for the past century. The first transatlantic telegraph cable was able to transmit a letter from queen Victoria to the president of the United States [2]. While this message allegedly took several hours to transmit, modern fiber-optic cables carry several terabits of data each second across more than $10{,}000\ \text{km}$ [3]. The need for cheap and reliable transmission capacity, both over short and long distances, is however expected to keep increasing for the foreseeable future [4]. This need motivates the search for ever-higher performance and at the same time more power efficient methods of performing data transmission.

Fiber-optic technologies have been rapidly replacing electrical connections on increasingly short distances, from the transcontinental down to links of only a few meters in length. Key enabling technologies for this have been the semiconductor laser [5–8] and the low-loss optical fiber [9]. More recently the erbium-doped fiber amplifier (EDFA) [10] has enabled data transmission over long distances without the need for repeater stations. The EDFA has the ability to amplify optical signals across a bandwidth of several THz around wavelengths close to $\lambda = 1550\ \text{nm}$. This wavelength region also corresponds to the lowest loss region in the silica fiber’s transparency window. To take advantage of the wide available transmission window, the concept of wavelength-division multiplexing (WDM) was introduced. WDM involves transmitting
several data channels in parallel through the same physical fiber by using different wavelengths, all of which fit within the gain bandwidth of the EDFA.

In today’s commercial WDM systems, an array of lasers, each operating at a slightly different wavelength, is used as light source. The laser light from each source is individually modulated with data after which they are recombined and transmitted down the link. A sketch of such a transmitter is shown in figure 1.1 (a). To help bring down the cost and power consumption of communications systems, various degrees of photonic integration have been developed over the past decades [11]. State-of-the-art demonstrations include complete multi-wavelength transmitters in indium phosphide [12], or complex multi-layer circuits in complimentary metal-oxide-semiconductor (CMOS)-compatible photonic circuits [13]. To further decrease the footprint of the transmitter and the control electronics, it is of interest to evaluate replacing the set of lasers with a single multi-wavelength light source. A chip-scale optical frequency comb, as shown in figure 1.1 (b), could provide this possibility.

Optical frequency combs in general are multi-wavelength light sources whose frequency lines are evenly spaced and phase-locked to each other. Looking at it from the time domain, this corresponds to a repetitive waveform with a repetition rate set by the frequency spacing of the comb lines. While mode-locked lasers qualitatively match this picture [14, 15], the first frequency combs where the central wavelength could be set freely were generated using electro-optic modulation of a continuous-wave (CW) laser tone [16]. These technologies have over the past decades enabled a large variety of communications demonstrations, including but not limited to Refs. [17–21]. More recently, it has been shown that combs with multi-GHz line spacing can be generated from a microresonator using a single pump laser [22, 23]. Microresonator combs made in a CMOS-compatible platform have the potential to allow co-integration of both optical and electrical components on a single chip [24]. This could revolutionize the availability of chip-scale multi-channel transceivers.
1.1 This thesis

The focus of this thesis is the usage of microresonator combs in the optical communications context. Previous work in this field indicate that microresonator combs hold great promise in enabling chip-scale multi-channel WDM data transceivers [25–27]. This thesis has focused on assessing their potential for long-haul communications and high-order modulation formats. An important distinction with respect to the quoted previous work is that we have focused on microresonators operating in the normal dispersion regime. This regime holds promises of high power conversion efficiency [28]. Ideally, a high power conversion efficiency be translated to a lowered pump power requirement or higher output comb line powers, ideally a combination of both!

Chapter 2 introduces the necessary concepts and performance metrics needed to understand and describe modern coherent optical communication systems. The chapter also gives a brief overview of the optical frequency comb technologies that have been employed and used in this field. In Chapter 3 the operational principle of microresonator frequency combs is described in detail. Following that, Chapter 4 will focus on and describe the stable frequency comb states that the microresonator systems can support. It will argue the pros and cons of each, with focus on the requirements for communication systems. Finally, in Chapter 5 we discuss what the near and mid-term future might hold. Following the appended papers, the Appendices will include schematics for code written in this project for the coherent receiver and for microresonator comb state simulations as well as a description of the stage setup used for the actual combs used in our lab.
Chapter 2

Optical frequency combs in coherent communications

2.1 Coherent communications systems

A generic point-to-point optical transmission system consists of a transmitter, the link itself and a receiver, see figure 2.1. Depending on the length of the link, it may consist of either a single span of fiber \( (N_A = 1) \), or multiple ones requiring periodic amplification \( (N_A > 1) \). For distances beyond a few kilometers, fiber-optic links today typically encode data on both the amplitude and phase of the optical field. By using both quadratures of the field, more data can be transmitted over the same bandwidth. To encode data on both quadratures (often called in-phase, I, and quadrature-phase, Q, in this context), a dual-drive Mach-Zehnder modulator (MZM) is typically used \([29]\). By programming the phases in the two arms carefully (denoted \( \phi_1(t) \) and \( \phi_2(t) \))

\[ \text{Data in} \rightarrow \text{Transmitter} \rightarrow \text{Link} \rightarrow \text{Receiver} \rightarrow \text{Data out} \]

Figure 2.1. Sketch of a periodically amplified optical transmission link. After the target data is encoded by the transmitter, the link consists of \( N_A \) fiber spans, each followed by an amplifier to compensate for the fiber loss. At the end, a receiver decodes the transmitted data.
2. CHAPTER. OPTICAL FREQUENCY COMBS IN COHERENT COMMUNICATIONS

Figure 2.2. A sketch of a single-polarization transmitter containing a dual-drive MZM. In a dual-polarization setup, the outputs from two such modulators are combined with a polarization beam combiner.

Figure 2.3. A sketch of a single-polarization coherent receiver showing the 90° hybrid with the 90° phase shifter and the balanced photodetectors. Extending this to a dual-polarization receiver requires putting a polarizing beam splitter and polarization rotator in front the signal input and duplicating the other components. The same LO can be used for both polarization channels at the cost of half its power.

in the equation below), both the amplitude and the phase of the final signal, $E_{\text{sig}}$, can be controlled independently:

$$E_{\text{sig}}(t) = \frac{E_{\text{laser}}}{2} \left( \exp(i\phi_1(t)) + i \exp(i\phi_2(t)) \right).$$

On the receiver side, to decode the data on both the phase and the amplitude of the optical field, a coherent receiver is required. In a coherent receiver, the incoming optical signal is mixed together with a free-running local oscillator (LO) in an optical 90° hybrid. At the output of the hybrid, a pair of balanced photodetectors are placed from which one can extract the real as well as the imaginary part of the down-mixed field. A sketch demonstrating this setup is shown in figure 2.3. Constellation diagrams for some common modulation formats are shown in figure 2.4. By increasing the number of discrete levels in the I and Q planes, one can encode more bits in each symbol going from 2 bits/symbol in QPSK to 8 bits/symbol in 256QAM.

Once the down-mixed field is recorded, digital signal processing (DSP) can be used to recover the transmitted signal by removing the relative frequency offset of the LO. Later components of the DSP can then compensate for various
2.2. WAVELENGTH-DIVISION MULTIPLEXING

Figure 2.4. Constellation diagrams displaying all valid symbols for QPSK, 16QAM, 64QAM, and 256QAM. Note that for equal power normalization, the distances between the valid constellation points decrease as the order goes up resulting in higher sensitivity to added noise.

Figure 2.5. (a) Sketch of a three-channel WDM transmitter. The lasers operate at three different wavelengths and are modulated individually. (b) Sketch of the optical spectrum after the transmitter. The aggregate bandwidth of the transmitter increases proportionally with the channel count.

other signal impairments using a complex-valued equalizer. By knowing the used modulation format, the recovered signal can then be translated to a bit sequence. A diagram describing the DSP used in Paper D is included in Appendix A.

2.2 Wavelength-division multiplexing

Another important component in an optical communications system is the fiber link itself. The Shannon-Hartley theorem states that the maximum theoretical capacity in bits/s, \( C \), of a Gaussian channel is proportional to the signal bandwidth, \( B \) [30]:

\[
C = B \log_2 \left( 1 + \frac{S}{N} \right),
\]

(2.2)

where \( S \) is the total received electrical signal power and \( N \) is the total received electrical noise power. While the capacity provided by Equation 2.2 only
corresponds to a theoretical upper limit for a Gaussian channel, it provides guidance in the scaling laws of the physical quantities involved also for a fiber link. By encoding data onto multiple wavelengths, the total bandwidth, $B$, over which data is transmitted is increased, while the signal-to-noise ratio (SNR) $S/N$ can remain constant leading to a linear increase in available capacity. The concept of WDM thus allows more data to be transmitted through the same physical fiber without requiring a corresponding increase in transmitter and receiver electrical bandwidth. Figure 2.5 provides a sketch of this concept.

At some point however, most real systems will reach a point where the effective bandwidth cannot be increased any further. This can be due to a lack of available matching components, such as amplifiers, or because there are other systems occupying neighbor frequencies. At that point the only way to increase the available capacity is to increase the SNR. In practice this also requires increasing the modulation format complexity. By encoding more bits of data into each transmitted symbol, the effective data rate will increase at the cost of higher sensitivity to the noise level. Section 2.3.1 will discuss in more detail how these requirements are connected.

## 2.3 Performance metrics and requirements

While the signal bandwidth provides one of the fundamental limits in communication systems, the SNR provides the other. In practice, measuring the electrical SNR of an optical channel is cumbersome as it requires receiving the signal and correcting for effects caused by the receiver. A simpler measurement, using an optical spectrum analyzer, can instead yield similar signal quality information through the optical signal-to-noise ratio (OSNR) metric:

$$\text{OSNR} = \frac{P_{\text{sig}}}{P_n},$$  \hspace{1cm} (2.3)

where $P_{\text{sig}}$ is the optical signal power in one channel and $P_n$ is the noise power measured over a specified bandwidth $\Delta \nu_{\text{OSA}}$ (typically 12.5 GHz or 0.1 nm). The OSNR metric can be translated to an electrical SNR under certain conditions \[31, 32\]. One has to assume that the data reception is performed using a coherent receiver and that the noise is dominated by additive white Gaussian noise (AWGN). This condition can typically be fulfilled by performing the OSNR measurement after the first amplifier in the link.

$$\text{SNR} = \text{OSNR} \frac{\Delta \nu_{\text{OSA}}}{B}. $$  \hspace{1cm} (2.4)
Here, $\Delta \nu_{\text{OSA}}$ denotes the bandwidth over which the optical noise floor was measured while $B$ denotes the channel bandwidth. A 25 GBd optical signal with 30 dB OSNR (measured using an optical spectrum analyzer at 0.1 nm resolution) will for example translate to a received electrical signal with 27 dB SNR. As OSNR lends itself to easy measurement, it is often used as a metric against which performance is compared. We will in the following sections talk about signal quality requirements in terms of minimum required OSNR, or signal impairments in terms of OSNR penalties.

The optical fiber is however also affected by intrinsic nonlinear effects. The optical power present in the fiber is directly (and deterministically) affecting the signal itself [33]. While numerically simulating such a nonlinear system is possible (see for example the propagation code described in Appendix B), predictions and estimations can be made without it. In a broadband WDM link (without optical dispersion compensation) where the data transmitted in each channel is uncorrelated, the nonlinear effects are noise-like [34]. They can therefore be modelled as such. This estimation allows calculating an effective OSNR after a link for each individual channel and is commonly known as the Gaussian noise model [35, 36]:

$$\text{OSNR}_{\text{eff}} \approx \frac{P_{\text{sig}}}{P_{\text{ASE, tot}} + \alpha P_{\text{sig}}^3}, \quad (2.5)$$

where $\alpha$ is a constant depending on the link parameters including the fiber nonlinearity, the link length, and the number of channels. $P_{\text{sig}}$ again corresponds to the power in a single channel, while $P_{\text{ASE, tot}}$ is the accumulated white noise from the link amplifiers [31]:

$$P_{\text{ASE, tot}} \approx N_A 2n_{\text{sp}} \Delta \nu \Delta \nu (G - 1). \quad (2.6)$$

In the above equation, $N_A$ denotes the number of amplifiers in the link while $2n_{\text{sp}}$, $\Delta \nu$ and $G$ denotes the amplifiers’ noise figure, the bandwidth over which the noise is measured, and the gain of each amplifier. Since the effective noise at the receiver is now also increasing with transmitted signal power, there will in practice be an optimal launched signal power for any given link configuration.

### 2.3.1 Modulation-format dependent requirements

As a final step to be able to pose an OSNR requirement on a transmitter, one also has to select a target bit error ratio (BER) for the link. While error-free operation is typically wanted, a low error ratio, such as $10^{-15}$ (corresponding
to one bit error every three hours in a 100 Gb/s link), is often cited as a target
for both research and commercial systems [37, 38]. For a system where
the noise is dominated by AWGN, the BER can be related to the received OSNR
according to [32]:

\[
 BER \approx \frac{2}{\log_2(M)} \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \sqrt{\frac{3}{2(M-1)} \frac{\Delta \nu_{OSA}}{B \text{OSNR}}} \right). \tag{2.7}
\]

This equation is valid for \( M \)-ary quadrature amplitude modulation (QAM)
formats under the assumption that Gray coding is used and that all symbol
errors are between adjacent symbols. To illustrate the noise sensitivity of
various common modulation formats Figure 2.6 shows the expected BER as a
function of received OSNR.

In practice, applying a forward error correction (FEC) code to the link
can be advantageous as it can allow significant relaxation of the received
OSNR requirement. These algorithms allow increasing the link’s BER target
at the cost of data overhead [39]. A pre-FEC BER target between \( 10^{-3} \)
and \( 10^{-2} \) is common among research papers (including appended papers A
and D) where overheads of around 10% are accepted. From Equation 2.7
(or Figure 2.6), this can now be translated to a final modulation-format
dependent minimum received OSNR requirement. To calculate a corresponding
transmitted OSNR requirement, the expected link noise has to be calculated.
Since the effective noise depends on the link length and the signal powers (as
described in equation 2.5), to make an example calculation, we have to make
some assumptions.
2.3. PERFORMANCE METRICS AND REQUIREMENTS

Figure 2.7. Required transmitted OSNR as a function of received OSNR for three different signal launch powers and two link lengths assuming fully linear propagation. The link represented by the fully drawn lines is assumed to contain a single EDFA with a noise figure of 5 dB operating at 20 dB gain while the dashed lines represent a link containing 10 such EDFAs. Note that by increasing the launch power we can make sure that the required transmitted OSNR stays similar to the received one.

Figure 2.8. (a) Effective OSNR as a function of channel signal power after a single fiber span according to equation 2.5. The nonlinear link has 10 (blue) or 100 (red) channels containing signals with 50 GBd symbol rate. The EDFA is assumed to have a noise figure of 5 dB while operating at 20 dB gain to compensate for 100 km of single-mode fiber transmission. (b) Required transmitted OSNR as a function of received effective OSNR when the system is operating at the optimum launch power for 1, 5 and 10 fiber spans.
In a system without fiber nonlinearities, it is of interest to note that there will be no additional OSNR requirements on the transmitter. To compensate for the accumulating noise from the link amplifiers, one can simply increase the transmitted signal powers. Figure 2.7 shows the required OSNR at the transmitter side for a linear link for some cases of link length and launch power.

In a more realistic link, affected by fiber nonlinearities according to equation 2.5, there exists an optimum launch power above which the effective OSNR will decay, see Figure 2.8(a). Figure 2.8(b) displays the required transmitted OSNRs for an estimated received effective OSNR. To reach higher received effective OSNRs than figure 2.8 allows for (i.e. > 35 dB), one has to use shorter span lengths (i.e. < 100 km) or distributed amplification. It is important to note that the transmitted OSNR requirements apply after data modulation, any light sources in the transmitter will therefore likely need additional margin depending on penalties caused by the connected components.

2.3.2 Phase noise

While lasers are considered monochromatic light sources, they do have a non-zero linewidth and suffer from phase noise. Similarly to the OSNR, the phase noise requirements on the laser source will be modulation format dependent as high order formats encode information with higher density in the phase. The effect of phase noise on a coherent communication systems’ BER has been under active investigation since the first coherent systems in the 80’s [40]. The wish for a single-number quantity with which to describe laser phase noise has led to common usage of the 3 dB Lorentzian laser linewidth metric, $\Delta \nu_L$. Often this is further normalized with respect to the symbol period: $\Delta \nu_L T_S$. Assuming a Lorentzian linewidth allows modelling the phase drift as a Wiener process where the drift between consecutive symbols is a zero-mean Gaussian random variable with variance $\sigma^2 = 2\pi \Delta \nu_L T_S$ [41, 42]. This permits the effect of a small amount of phase noise on the carrier to be translated to an OSNR penalty [42]. Putting it the other way, by accepting a certain OSNR penalty (e.g. 1 dB), one can estimate the required laser linewidth for a system. Figure 2.9 shows simulated trends for how the BER increases from an operating point of $10^{-3}$ as the laser linewidth is increased. Accepting a 1 dB penalty puts a clear limit on what amount of phase noise is accepted on the laser. In a real system however, one would probably pick the lasers in such a way that their linewidth does not dominate the BER (i.e. keeping the linewidths below where the curves start growing).

While a 3 dB linewidth provides a good metric with which to simulate lasers and to estimate DSP performance, it does not necessarily tell the whole picture.
2.4. OPTICAL FREQUENCY COMB TECHNOLOGIES

The phase noise statistics of lasers have been known for some time not to be fully Lorentzian [43]. Typically it is the longer-term drift of the phase noise that doesn’t conform [44]. At first guess, this will not significantly impact high symbol rate data transmission as the noise remains Lorentzian in the frequency region where the blind phase-tracking component of the DSP operates. This is however only true given that the non-Lorentzian noise component is weak and does not affect frequency regions of interest [45]. Of additional concern when it comes to optical frequency combs is the presence of any accumulating phase noise as one looks at comb lines far away from the center. Four-wave mixing tends to scale any uncorrelated phase noise in an unfavorable manner [46] while combs based on electro-optic modulation also show a small amount of scaling [47]. While scaling in microresonator comb sources is still an open research topic, in Paper B we performed measurements on one device to verify that any scaling stays within an order of magnitude. By selecting a sufficiently narrow linewidth pump laser, one can thereby ensure that all comb lines can be used for data communications. For guidance, Table 2.1 contains received and transmitted OSNR and linewidth requirements for a set of common modulation formats and a single-span 100 km WDM link.

2.4 Optical frequency comb technologies

In the context of optical communications, optical frequency combs are currently treated as black-box multi-wavelength coherent light sources. While this is certainly a useful view, combs have wider use cases and properties that are
2. CHAPTER. OPTICAL FREQUENCY COMBS IN COHERENT COMMUNICATIONS

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<tr>
<td></td>
<td>100</td>
<td>31.6</td>
<td>1000</td>
<td>50</td>
<td>36.1</td>
</tr>
<tr>
<td>256QAM</td>
<td>25</td>
<td>31.4</td>
<td>75</td>
<td>200</td>
<td>36.3</td>
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<td></td>
<td>50</td>
<td>34.4</td>
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<td></td>
<td>100</td>
<td>37.4</td>
<td>300</td>
<td>50</td>
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</table>

Table 2.1. Table showing the OSNR and linewidth requirements for different modulation formats with a target BER of $10^{-3}$. The OSNR_{Rx} value is calculated using equation 2.7. The linewidth requirement is set to allow 0.2 dB OSNR penalty (BER = $1.2 \times 10^{-3}$ according to figure 2.9). The required transmitted OSNR_{Tx} includes the 0.2 dB penalty from the linewidth and is further assuming 100 km single-span transmission operating at optimum launch power with dense WDM, spanning 5 THz where the receiver has an EDFA with 5 dB noise figure (calculated the same way as in figure 2.8).

worth exploring.

As stated in the introduction, the first optical frequency comb sources were the electro-optic combs [16]. By modulating a laser externally using a phase modulator, one can set the resulting comb line spacing independently of the size of the laser cavity. Modern electro-optic frequency combs, using several cascaded phase modulators, can generate combs with more than 50 high-powered lines [48].

The bandwidth of a frequency comb can be significantly extended by amplification and successive broadening in a nonlinear medium. While nonlinear broadening in glass was observed already in the end of the 60’s [49], so called supercontinuum generation spanning more than an octave was first reported using photonic crystal fibers and a mode-locked laser in 2000 [50]. By having access to an octave of bandwidth, the low-frequency part of the comb can be doubled and beat with the higher-frequency lines. This allows referencing and stabilizing the frequency comb with an external radio frequency (RF) reference [51]. This self-referencing technique has now also been demonstrated using broadening in highly nonlinear fiber with mode-locked lasers [52, 53], electro-optic combs [54, 55] and more recently with microresonator combs [56].
2.4. OPTICAL FREQUENCY COMB TECHNOLOGIES

Stabilized optical frequency combs have been demonstrated to allow advances for a wide range of applications. Self-referenced combs permit measuring optical frequencies with the accuracy of RF references [57–59]. The technique of using combs as rulers [60, 61], against which calibrations can be made resulted in the physics Nobel prize of 2005. Applications include molecular spectroscopy [62–65], astronomy [66, 67] reaching all the way to attosecond scale measurements [68]. By using two combs in parallel, spectroscopic measurements of a sample’s phase response also becomes practical [69–73]. Combs also allow for precise distance measurements on the nanometer scale using both time-of-flight [74] as well as interferometric [75, 76] techniques.

By detecting the pulse train from a frequency comb in a photodiode, combs can effectively be used as microwave synthesizers [77–80]. Furthermore, adjusting the frequency comb lines’ relative amplitudes and phases allows for precise control of the pulse train’s time domain shape. By combining a frequency comb with pulse shaping optics, one can thereby implement arbitrary microwave waveform generation [81, 82].

Not all applications require absolute frequency stability however. For applications such as optical communications, where a handful of relatively stable lines can be enough, comb sources producing fewer lines might make more sense. Integrated platforms provide a pathway to reduce the bulkiness of the comb sources, usually at the cost of line power or bandwidth. This can be interesting in applications where phase stability, power consumption or space availability provide critical constraints. Both electro-optic combs [83] and mode-locked lasers [84, 85] have been demonstrated in various integrated platforms. Integrated options however also include a new comb type: the microresonator combs [86]. In 2007, comb generation using whispering-gallery mode resonators consisting of silica-based toroidal microcavities was demonstrated [23]. Demonstrations in several other material platforms quickly followed including using CMOS-compatible planar silicon nitride rings [24, 87–89]. The working principle of these devices is the nonlinear Kerr effect [90]. Microresonator combs are usually pumped with a single or a few high-powered laser lines, which then through four-wave mixing cascade into a full comb. Assuming that the combs’ line powers are high enough to meet the OSNR requirements discussed in the previous section, the combination of single-laser pumping and multi-GHz line spacing provide an attractive platform for multi-channel coherent communications experiments. The dynamics and function of these systems (including their soliton-like characteristics [89, 91]) is explored in more detail in the next chapter.
2.5 Combs as WDM light sources

By removing the need for multiple free-running lasers, mode-locked lasers and electro-optic frequency combs simplified early laboratory-based on-off keying (OOK) WDM scheme demonstrations [92]. A single electro-optic modulator fed with an RF oscillator doesn’t yield a comb of significant bandwidth however. Demonstrations overcame this bandwidth limitation using nonlinear broadening [92]. By the year 2000, this allowed mode-locked laser-based sources to be used for the generation of over 1000 data channels [17]. A few years later, similarly broadened electro-optic combs were tested in a real-world link of about 100 km [18].

As discussed in Section 2.3, coherent modulation formats pose more strict requirements on the light source in terms of available OSNR and phase noise. Electro-optic combs have in the past few years been optimized to allow for increasingly high order formats, going from QPSK [93] to 16QAM [19, 94], 64QAM [20], and 128QAM [21]. While the spectral envelope of the broadened electro-optic combs can be kept flat [95], the phase noise of electro-optic combs scales quadratically as one moves away from the central wavelength [47]. This effect leads to varying OSNR penalties across the comb [19]. A way of minimizing this penalty is to instead modulate different formats on the carriers depending on their qualities (in terms of both OSNR and phase noise) [96]. Alternatively, one can elect to only use the highest quality lines in the comb, thereby pushing the modulation formats up to 256QAM [97].

The demonstrations quoted above all rely on a frequency comb providing a stable multi-wavelength light source replacing a rack of lasers. By only having to purchase a single low-linewidth laser, certain scenarios might this way allow for modulation formats with stricter linewidth requirements than what would otherwise be possible. To extract further gains from having a comb source will however require exploiting some of its unique coherence properties. By knowing in advance the exact frequency spacing of the data channels, it is possible to pre-compensate the signal in multiple channels for deterministic nonlinear effects caused by the fiber link [98, 99]. While the single-channel precompensation results in small gains also in laser-array based systems [100], comb-based few-channel scenarios have been shown to allow a doubling [101] or even tripling [102] of the transmission distance. Apart from enhancing nonlinearity-precompensation, stability of the channel spacing can be used to minimize the guard bands between adjacent channels, thus increasing the net spectral efficiency of broadband superchannels [21, 103, 104]. Additionally, comb lines are not only frequency-stable but also phase coherent. Recent demonstrations indicate that this phase coherence allows joint tracking of the
The final topic of the frequency comb-based WDM source chapter is that of integration. A complete chip-scale transmitter where the comb generation and the data modulation can happen on the same chip has been a target which researchers have worked towards for some time. Current demonstrations typically contain a partially integrated comb source while the data transmitter remains set up using external discrete components. Initial integrated comb demonstrations using OOK modulation were done using quantum-dash mode-locked lasers [106] with microresonator comb demonstrations coming a few years later [107, 108]. Coherent modulation formats followed with quantum-dash mode-locked lasers enabling QPSK [109] up to 32QAM [110] modulation. Other integrated technologies include silicon-organic hybrid modulators [111] (note that electro-optic combs still require an external RF source in addition to the pump laser). Of main interest to this thesis are however the microresonator combs. Recent demonstrations include coherent communications over short distances [25, 26] in some cases with more than 100 comb lines [27]. Here, Paper D reaches all the way to PM-64QAM modulation with 20 lines. Paper A additionally proves that using QPSK and 16QAM long-haul links are also possible. These demonstrations prove that integrated comb sources, particularly microresonator combs, are now reaching a level where they can power modern coherent data communication links.
2. CHAPTER. OPTICAL FREQUENCY COMBS IN COHERENT COMMUNICATIONS
Chapter 3

Microresonator dynamics

The microresonator term is typically used to describe a millimeter-scale resonant cavity. In this chapter we will discuss planar microresonators, but the models are similar for Fabry-Perot cavities, microspheres, and other optical microcavity systems as well. Key features of microresonators are their small sizes (permitting resonance to be spaced by several tens of gigahertz) and their low roundtrip losses (corresponding to high quality factors). While optical microcavities can be used for a large array of topics [112], much of their functionality is centered around filtering and resonantly enhancing signals.

3.1 Microresonators in the linear regime

To describe the operating principle of microresonators, we first have to describe their linear, low-power operation. A microresonator is in the most basic view a two-component system: a directional coupler and a looped waveguide. Figure 3.1 shows a sketch where a second directional coupler (a drop port) has also been added. The complex electric fields at some notable positions has been indicated.

The coupling of the fields between the different paths can be described
using the following equations [113–116]:

\[
\begin{align*}
\begin{bmatrix} E_{\text{through}} \\ E_2 \end{bmatrix} &= \begin{bmatrix} t & \kappa \\ -\kappa^* & t^* \end{bmatrix} \begin{bmatrix} E_{\text{in}} \\ E_5 \end{bmatrix}, \\
\begin{bmatrix} E_{\text{drop}} \\ E_4 \end{bmatrix} &= \begin{bmatrix} t_d & \kappa_d \\ -\kappa_d^* & t_d^* \end{bmatrix} \begin{bmatrix} 0 \\ E_3 \end{bmatrix},
\end{align*}
\]

\(E_3 = \sqrt{a} \exp \left( i \beta \frac{L}{2} \right) E_2,\) \hspace{1cm} (3.3)

\(E_5 = \sqrt{a} \exp \left( i \beta \frac{L}{2} \right) E_4.\) \hspace{1cm} (3.4)

The two coupling regions are defined by the constants \(t, \kappa, t_d,\) and \(\kappa_d,\) such that \(|t|^2 + |\kappa|^2 = 1\) and \(|t_d|^2 + |\kappa_d|^2 = 1.\) The ring waveguide loss is defined as \(a = \exp \left( -\frac{\alpha}{2} L \right)\) where \(\alpha\) is the power propagation loss per unit length and \(L\) is the physical length of the resonator. The propagation constant is denoted as \(\beta.\) The power out at the through and drop port can then directly be derived:

\[
|E_{\text{through}}|^2 = |E_{\text{in}}|^2 \frac{|t|^2 + a^2|t_d|^2 - 2a|t||t_d| \cos (\beta L - \phi_t - \phi_{t,d})}{1 + a^2|t|^2|t_d|^2 - 2a|t||t_d| \cos (\beta L - \phi_t - \phi_{t,d})},
\]

\(|E_{\text{drop}}|^2 = |E_{\text{in}}|^2 \frac{a|\kappa|^2|\kappa_d|^2}{1 + a^2|t|^2|t_d|^2 - 2a|t||t_d| \cos (\beta L - \phi_t - \phi_{t,d})}.\) \hspace{1cm} (3.5)

The transmission coefficients have been expanded according to \(t = |t| \exp (i\phi_t)\) and \(t_d = |t_d| \exp (i\phi_{t,d}).\) The case where there is no drop port can be trivially
extracted by setting \( t_d = 1 \) and \( \kappa_d = 0 \):

\[
|E_{\text{through,nodrop}}|^2 = |E_{\text{in}}|^2 \frac{|t|^2 + a^2 - 2a|t| \cos(\beta L - \phi_t)}{1 + a^2|t|^2 - 2a|t| \cos(\beta L - \phi_t)}.
\]

(3.7)

The above relations (particularly equation 3.5 and equation 3.7) will be useful for further derivations as they describe the system’s spectral response.

### 3.1.1 Inter-waveguide coupling

The coupling parameter \( \kappa \) between two adjacent waveguides is naturally dependent on the interaction length and the mode overlap between the waveguides’ transverse modes [117]:

\[
\kappa_{12} \approx \omega \epsilon_0 \int (n_1^2 - n_{cl}^2) e_{t,1}^* \cdot e_{t,2} dx dy,
\]

(3.8)

where the tangential electric fields \( e_t \) are power normalized. The electric field values for the transverse modes of interest can be simulated using a standard mode solver. For well confined modes, the mode overlap will be low, leading to a weak coupling unless the interaction length is extended accordingly. The easiest way to increase the coupling interaction length is to make the bus waveguide follow the resonator over a part of a round-trip in a “pulley” coupling scheme [118] yielding an larger effective coupling parameter:

\[
\kappa \approx \sin(\kappa_{12} L_{\text{pulley}}).
\]

(3.9)

In Paper D, this model permits motivating that power coupling parameters on the order of a few percent are achievable.

### 3.1.2 System characterization

To probe the system, we can make use of the fact that \( \beta \) depends on the probe laser frequency. This allows information about \( a \) and \( |t| \) to be extracted from a sample by performing a scan with a tunable CW laser. Figure 3.2 shows an example of how such a scan should look like for a resonator without a drop port. Apart from the minimum power inside the resonance, it is possible to measure the resonance’s FWHM. In the general case, the half maximum is defined as the point where the throughput power is exactly half-way between the minimum and the maximum. The measured \( \Delta \omega_{\text{FWHM}} \) can be translated to a \( \Delta \beta_{\text{FWHM}} \) according to:

\[
\Delta \beta_{\text{FWHM}} \approx \Delta \omega_{\text{FWHM}} \frac{\partial \beta}{\partial \omega} \approx \Delta \omega_{\text{FWHM}} \frac{n_g}{c}.
\]

(3.10)
3. CHAPTER. MICRORESONATOR DYNAMICS

Figure 3.2. (a) Simulated scan for an undercoupled microresonator with the following parameters: \( a^2 = 0.98 \), \( L = 2\pi 100 \mu m \), \( |t|^2 = 0.95 \), \( |t_d|^2 = 1 \), \( \phi_t = \phi_{t,d} = 0 \), \( \beta = n_{\text{eff}} \frac{2\pi}{\lambda} \), \( n_{\text{eff}} = 1.5 \). The extinction ratio, \( P_R \), and the free spectral range (FSR) have been highlighted. (b) Zoomed-in view of one of the resonances. The resonance full width at half maximum (FWHM) has been highlighted.

Meanwhile, the expected \( \Delta \beta_{\text{FWHM}} \) can be extracted from equation 3.5 [116]:

\[
\Delta \beta_{\text{FWHM}} \approx 2 \frac{1 - a|t||t_d|}{L \sqrt{a|t||t_d|}}.
\]  

Setting equations 3.10 and 3.11 to equal yields the following relation between the FWHM and the losses:

\[
\Delta \omega_{\text{FWHM}} \approx \frac{2c}{n_g L} \frac{1 - a|t||t_d|}{\sqrt{a|t||t_d|}}.
\]  

To disentangle the contributions of the propagation loss, \( a \), and the coupling loss, \( |t| \), one can also look at the extinction ratio for the through port:

\[
P_R = \frac{|E_{\text{through, max}}|^2}{|E_{\text{through, min}}|^2} = \frac{(1 - a|t||t_d|)^2}{(a|t||t_d| - |t|)^2} \approx \frac{(1 - a|t||t_d|)^2}{(a|t||t_d| - |t|)^2}.
\]

By measuring \( \Delta \omega_{\text{FWHM}} \) and the extinction ratio, \( P_R \), we can then, using equations 3.12 and 3.13, numerically extract the two parameters: \( |t| \) and \( a|t_d| \).
To accurately convert between the resonance FWHM and the loss parameters, we need to know the group index, \( n_g \), of the material. Fortunately, we can figure that out by finding the frequencies of two adjacent resonances and calculating the FSR: \( \Delta \omega_{\text{FSR}} = |\omega_1 - \omega_2| \). We know that \(|\beta(\omega_1)L - \beta(\omega_2)L| = 2\pi\). By assuming that the group index is identical for adjacent resonances (this assumption might not be valid for resonators with strong chromatic dispersion and large FSR!), we can extract:

\[
\frac{2\pi c}{\Delta \omega_{\text{FSR}} L} 
\]

(3.16)

It is now possible to numerically solve equation 3.14 and 3.15 for \( a \) and \(|t|\). Note that the symmetry in the denominator of equation 3.15 will result in a pair of symmetrical solutions however. To distinguish an undercoupled resonator from an overcoupled one, further information is needed. This can be either through prior knowledge based on the system design or by measuring the phase response of the transmission scan.

### 3.1.3 Chromatic dispersion

As will become clear later in this chapter, the magnitude and the sign of the resonator waveguide’s chromatic dispersion is also of great interest. One of the effects caused by the presence of chromatic dispersion is that the resonator’s FSR becomes frequency dependent. We can use this knowledge to characterize it. By measuring the locations of three adjacent resonances, \( \omega_{-1}, \omega_0, \) and \( \omega_1 \) (similar to figure 3.2), we can write the following equations:

\[
\beta(\omega_0)L - \beta(\omega_{-1})L = 2\pi, \quad (3.17)
\]

\[
\beta(\omega_1)L - \beta(\omega_0)L = 2\pi. \quad (3.18)
\]

After Taylor-expansion according to:

\[
\beta(\omega_{\pm 1}) = \beta(\omega_0) + (\omega_{\pm 1} - \omega_0) \frac{\partial \beta}{\partial \omega} \bigg|_{\omega=\omega_0} + (\omega_{\pm 1} - \omega_0)^2 \frac{1}{2} \frac{\partial^2 \beta}{\partial \omega^2} \bigg|_{\omega=\omega_0}, \quad (3.19)
\]

we can directly extract the second order dispersion parameter:

\[
\beta_2 = \frac{\partial^2 \beta}{\partial \omega^2} \bigg|_{\omega=\omega_0} \approx \frac{2\pi \Delta \omega_{\text{FSR}}}{L \bar{\omega}_{\text{FSR}}^3}, \quad (3.20)
\]

where \( \Delta \omega_{\text{FSR}} \) is the difference between the two adjacent FSRs and \( \bar{\omega}_{\text{FSR}} \) is the average FSR. For a \( \bar{\omega}_{\text{FSR}} \approx 2\pi \times 100 \text{ GHz} \) resonator, we need better than
2π × 1 MHz resolution in the Δω_{FSR} measurement to get β_2 with 100 ps^2/km resolution however. To allow averaging (or extraction of higher order dispersion terms), one should measure a larger set of resonance locations symmetrically around ω_0. By performing a polynomial fit for all resonances to the following simple Taylor expansion, one can extract the dispersion parameters with increased precision:

\[(\omega_\mu - \omega_0)\beta_1 + (\omega_\mu - \omega_0)^2\frac{\beta_2}{2} + (\omega_\mu - \omega_0)^3\frac{\beta_3}{6} + \ldots = \mu\frac{2\pi}{L}. \quad (3.21)\]

In the above equation, μ is an integer describing the relative resonance number.

### 3.1.4 Quality factors

A common metric to characterize resonators in general is to calculate their Q-factors. This metric describes how many oscillations the electric field makes in the resonator before decaying (because of absorption or outcoupling). While this metric is in principle defined in the time domain, for high-Q resonators it relates to the FWHM of its resonances according to [116]:

\[Q \approx \frac{\omega}{\Delta\omega_{FWHM}}. \quad (3.22)\]

When comparing different resonators to each other, it might be of interest to separate the loss effects caused by absorption and by outcoupling. One can in that case talk about *intrinsic* and *extrinsic* Q-factors. If a and |t| have already been measured, equation 3.14 can estimate what the Δω_{FWHM} would be for a resonator either without propagation loss or without coupling to the environment. From that estimated resonance width, the following Q-factors can be calculated:

\[Q_i \approx \frac{\omega n_g L \sqrt{a}}{2c(1-a)} \approx \frac{\omega n_g}{ca}, \quad (3.23)\]

\[Q_e \approx \frac{\omega n_g L \sqrt{|t|}}{2c(1-|t|)}. \quad (3.24)\]

For systems with low losses (i.e. high Q-factors), it can be shown that \(Q^{-1} \approx Q_i^{-1} + Q_e^{-1}\). Note that the Q-factors are proportional to the probing frequency, ω, as well as the resonator roundtrip length, L. Since both loss parameters (a and |t|) are defined per roundtrip, increasing both the frequency and the resonator length will proportionally increase the number of permitted electric field oscillations. When the losses are instead defined per unit length (as with α), the length-dependence goes away.
3.1.5 Optical confinement

The $Q$-factors are often mentioned in publications (including in the appended papers) as decreased losses correspond to longer (i.e. stronger) interactions. This can be understood by calculating the field intensity inside the resonator system. The relative intracavity power, or the on-resonance magnification, $M$, can be derived from equations 3.1 and 3.2 [114]:

$$M = \frac{|E_2|^2}{|E_{in}|^2} = \frac{1 - |t|^2}{(1 - a|t||t_d|)^2}. \quad (3.25)$$

For large $Q$-factors (i.e. when $a$ and $|t|$ are close to 1), $M$ is proportional to $Q$, meaning that measuring the $Q$-factor can give an estimation for the allowed intensity buildup inside the ring. Additionally it is worth noting that by decreasing the losses (increasing the $Q$-factor), the effective length along which interactions can occur is also increased. This will become important when considering nonlinear interactions later, as their strength will therefore be magnified quadratically with increasing $Q$.

3.1.6 Critical coupling

One potentially interesting special case is the situation of critical coupling. Critical coupling occurs when the coupling losses at the input are equal to the rest of the losses in the cavity:

$$|t| = a|t_d|. \quad (3.26)$$

In this situation, at resonance, equations 3.5 and 3.6 become:

$$|E_{through}|^2 = 0, \quad (3.27)$$

$$|E_{drop}|^2 = |E_{in}|^2 \frac{|t|}{|t_d|} \frac{1 - |t_d|^2}{1 - |t|^2}. \quad (3.28)$$

The ring absorbs the full power from the pump leaving nothing at the through port. If the resonator has a drop port, we can see from equation 3.26 that to maintain the critical coupling the propagation losses have to be decreased at the same rate as the coupling to the drop port is increased. In the special case of $|t_d| = |t|$, the absorption losses have to become zero! In this case the output power at the drop port, according to equation 3.28 will be equal to the input: $|E_{drop}|^2 \xrightarrow{t_d \rightarrow t} |E_{in}|^2$. The response of such a system is shown in Figure 3.3.
3.2 Nonlinear propagation

While equations 3.1–3.4 describe the filtering characteristics of resonators, to be able to model frequency comb states, the devices’ nonlinear behavior has to be taken into account. Light-matter interaction in general is governed by Maxwell’s equations. Their full analysis is beyond the scope of this thesis but can be found in reference [33]. They predict that for high optical intensities, such as the ones we will encounter inside the rings, the material will allow for nonlinear interactions between photons of different wavelengths. The part that concerns us the most is the interaction facilitated by the third-order susceptibility, $\chi^{(3)}$, causing the effective index of the mode in the material to be intensity-dependent [33]:

$$n(I) = n_{\text{eff}} + n_2 I(t).$$

(3.29)

To analyze waves propagating through a fiber or a waveguide, it is typically enough to model the field envelopes, $A$, rather than the electric field itself, where the amplitude is normalized such that $|A|^2 = I$. With this assumption, one can derive the nonlinear Schrödinger equation (NLSE) for the propagating waves [33]:

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A + i \sum_{k \geq 2} \frac{\beta_k}{k!} \left(i \frac{\partial}{\partial t}\right)^k A + i \gamma |A|^2 A.$$

(3.30)

Here, $\alpha$ denotes the fiber power losses per unit length while $\beta_k$ denotes the $k$-th coefficient in the Taylor expansion of the frequency-dependent propagation
constant of the material. The nonlinear parameter, $\gamma$, is related to the intensity-dependent refractive index, $n_2$, and the effective mode area, $A_{\text{eff}}$:

$$\gamma = \frac{\omega_0 n_2}{c A_{\text{eff}}}.$$  \hspace{1cm} (3.31)

The value of $A_{\text{eff}}$ can be simulated both for fibers and waveguides (also for non-transverse electromagnetic modes [119]) allowing estimations of the $\gamma$ parameter. For materials where tabulated data is scarce or inconsistent, this gives enough confidence in the order of magnitude to allow system simulations.

### 3.3 The Ikeda map

By modifying equations 3.1–3.4 to include the NLSE from equation 3.30, the nonlinear dynamics of the complete system can be explored. From here on, to simplify the analysis, we will focus on systems without a drop port, as shown in Figure 3.4. The transmission over one coupling region is further assumed not to change the phase: $\kappa = -\sqrt{\theta}$ and $t = \sqrt{1-\theta}$. Since we’re working with the field envelope in the NLSE, the phase evolution over one roundtrip will not be present. A corresponding relative phase offset, $\phi$ is therefore included in the equation:

$$\begin{bmatrix} A_{\text{through}} \\ A_2 \end{bmatrix} = \begin{bmatrix} \sqrt{1-\theta} & -\sqrt{\theta} \\ \sqrt{\theta} & \sqrt{1-\theta} \end{bmatrix} \begin{bmatrix} A_{\text{in}} \\ A_3 \end{bmatrix},$$  \hspace{1cm} (3.32)

$$A_3 = \exp(-i\delta_0) \int_0^L \frac{\partial A(z,t)}{\partial z} \, dz,$$  \hspace{1cm} (3.33)

where $A(0,t) = A_2$.

The phase offset, $\delta_0$, corresponds to the relative phase shift of the wave compared to the phase of a wave at a resonance wavelength: $\delta_0 = -\Delta \beta L \approx (\omega_{\text{mode}} - \omega_{\text{pump}}) \frac{L \mu_g}{c}$ and is applied at the end of each roundtrip.

This system model is sometimes called an Ikeda map after Ref. [120]. It allows simulations to be conducted using standard NLSE solvers (such as the split-step method described in Ref. [33]) by iterating the coupling region and the propagation element [121, 122]. As new pump light is coupled in at each roundtrip, it also allows correct noise handling [121]. The simulations included in Paper D were performed using this method. Appendix B contains a more detailed description of the algorithm.
3.4 The Lugiato-Lefever equation

While using the Ikeda map method to run system simulations works well, it is challenging to work with the model analytically. To get a single closed form expression that we can derive understanding from, we will have to make some additional assumptions and simplifications. We will assume that the field envelope barely changes over one roundtrip: \( A(L, t) \approx A(0, t) + L \frac{\partial A}{\partial z} \). The equations governing the field inside the resonator cavity can then be rewritten as:

\[
A_2 = \sqrt{\theta} A_{\text{in}} + \sqrt{1 - \theta} A_3, \tag{3.34}
\]
\[
A_3 = \exp(-i\delta_0) \left( A_2 + L \frac{\partial A}{\partial z} \right). \tag{3.35}
\]

Assuming that we have a weakly coupled resonator pumped close to a resonance, both \( \theta \) and \( \delta_0 \) are small. We can then Taylor expand them according to:

\[
\sqrt{1 - \theta} \approx 1 - \frac{\theta}{2}, \tag{3.36}
\]
\[
\exp(-i\delta_0) \approx 1 - i\delta_0. \tag{3.37}
\]

Inserting this back into equations 3.34 and 3.35 and linearizing in terms of the variables \( \theta \), \( \delta_0 \), and \( \frac{\partial A}{\partial z} \) gives us an expression for the intracavity field after roundtrip \( m \):

\[
A_2^{(m)} \approx \sqrt{\theta} A_{\text{in}} + A_2^{(m-1)} \left( 1 - \frac{\theta}{2} - i\delta_0 \right) + L \frac{\partial A}{\partial z}. \tag{3.38}
\]
3.5 BI-STABILITY AND COMB INITIALIZATION

We can now insert the NLSE from equation 3.30. Since we know that the change between consecutive roundtrips is small, we can write a differential equation describing the slow time-evolution of the wave at this point assuming a roundtrip time of $t_r$ [123, 124]:

$$\frac{\partial A}{\partial \tau} \approx \frac{A^{(m)} - A^{(m-1)}}{t_r}$$

$$= \frac{1}{t_r} \left[ \left( -\frac{L\alpha + \theta}{2} - i\delta_0 + iL \sum_{k \geq 2} \frac{\beta_k}{k!} \left( i \frac{\partial}{\partial t} \right)^k + i\gamma L |A|^2 \right) A + \sqrt{\theta} A_{in} \right].$$

(3.39)

This equation is generally known as the Lugiato-Lefever equation or the driven-and-damped nonlinear Schrödinger equation. It has been used to describe ring cavities of fibers and other Kerr media and since the early 90’s [125, 126]. It has more recently also been shown to be practical for modelling and simulating microresonator systems [124, 127]. As long as the pump detuning is not too large and the coupling strength is not too strong (i.e. while equations 3.36 and 3.37 are valid), it remains a powerful tool for analyzing the system dynamics. The Lugiato-Lefever model can be enhanced by including further physical effects including Raman scattering [128, 129], self-steepening [130], second order susceptibility [131–133] as well as thermal shifts [134, 135].

### 3.5 Bi-stability and comb initialization

Using the Lugiato-Lefever model from equation 3.39, it is then possible to analyze the behavior of microresonator systems. A perturbation analysis around a CW steady-state gives useful information about the system’s dynamics when pumping with a single CW pump [121, 125, 126, 136–138]. The CW steady-state, $A_s$, can be found by setting all time derivatives in equation 3.39 to zero:

$$0 = \left( -\frac{L\alpha + \theta}{2} - i\delta_0 + i\gamma L |A_s|^2 \right) A_s + \sqrt{\theta} A_{in} \Rightarrow$$

(3.40)

$$\theta |A_{in}|^2 = |A_s|^2 \left( \left( \frac{L\alpha + \theta}{2} \right)^2 + \delta_0^2 \right) - |A_s|^4 2\delta_0 \gamma L + |A_s|^6 \gamma^2 L^2.$$  

(3.41)

Depending on the pumping region, equation 3.41 has between one and three solutions for $|A_s|^2$. Figure 3.5 shows the steady-state solutions as the detuning,
3. CHAPTER. MICRORESONATOR DYNAMICS

\[ \delta_0, \text{ and the pump power, } P_{\text{pump}} = |A_{\text{in}}|^2, \text{ are varied. Note that the curve depicting the intracavity power as a function of detuning in figure 3.5(a) turns into a Lorentzian when } \gamma = 0. \text{ In that system, the FWHM matches the value for } \Delta \beta L \text{ extracted in the previous section (equation 3.11). It is furthermore worth noting that only the top and bottom CW solutions are stable (similarly corresponding to the top and bottom branch in figure 3.5(b)) [123].}

While a more complete analysis of the system behavior is presented in Refs. [137, 138], it is of interest to take a quick look at what is required to de-stabilize a CW solution. Any modulation instability (MI)-like behavior is likely key to being able to generate a comb state from an empty cavity with only a CW pump laser. Choosing the following ansatz allows for finding MI gain regions [125, 136]:

\[ A = A_s + A_{-1}(\tau) \exp(-i\Delta \omega t) + A_1(\tau) \exp(i\Delta \omega t). \]  
(3.42)

Under the assumption that \( |A_{-1,1}| \ll |A_s| \), the system experiences gain at frequencies offset from the pump, \( \Delta \omega \), according to:

\[
G(\Delta \omega) = \frac{1}{t_r} \left( -\frac{L \alpha + \theta}{2} + \sqrt{\gamma^2 L^2 |A_s|^4 - \left( 2\gamma L|A_s|^2 - \delta_0 + L \sum_{k \geq 2, \text{even}} \frac{\beta_k}{k!} \Delta \omega^k \right)^2} \right).
\]  
(3.43)

Important to note is that for there to be net gain, the term under the square root has to remain positive and be large enough to compensate for both the propagation losses and the coupling losses. Figure 3.6 shows an example gain spectrum. For the gain to be maximized at a certain frequency offset, \( \Delta \omega \), there will therefore be requirements on the physical parameters:

\[ 2\gamma L|A_s|^2 - \delta_0 + L \sum_{k \geq 2, \text{even}} \frac{\beta_k}{k!} \Delta \omega^k = 0. \]  
(3.44)

This means that either negative even dispersion orders or a positive \( \delta_0 \) is needed to achieve MI-based gain. Typically, the second order dispersion, \( \beta_2 \), is dominating this term, but examples have been shown where the presence \( \beta_4 \) makes a significant difference [139]. To avoid the global negative effective \( \beta_2 \) requirement one can design the waveguide so that the dispersion becomes anomalous only locally around the pump wavelength. In multi-mode waveguides the geometry can be designed so that higher order modes couple to the
3.5. BI-STABILITY AND COMB Initialization

Figure 3.5. Bistability curves showing the intracavity power as a function of (a) the detuning, and (b) the pump power. (c) Contour plot showing the number of CW steady-state solutions for each combination. In all cases the rest of the parameters are fixed according to: $\alpha = 0.1 \text{ dB/cm}$, $L = 2\pi \times 100 \mu m$, $\theta = 0.01$, and $\gamma = 2 W^{-1} m^{-1}$, which are realistic for Si$_3$N$_4$-based microresonators.
pumped mode at some wavelength leading to a locally strong perturbation of the dispersion [91, 140–144]. Figure 3.7 shows how this would look like for two coupled modes. The resonators used in Papers A-D are enabled using such an interaction. In these circumstances the comb is initialized using the degenerate four-wave mixing (FWM) process that was described above, while the new lines are then grown using non-degenerate FWM (that has no requirements on the sign of the dispersion).
3.6 Thermal effects and detuning

The refractive index of most materials is temperature-dependent. Since microresonators are typically pumped with high-powered laser light, the temperature of chips under operation is expected to be higher than room temperature. When tuning a pump laser into a resonance, more and more power gets coupled into the ring, thus increasingly heating it up. This causes a red-shift of the resonance for most material platforms, including silicon nitride [145]. Since the resonances shift towards longer wavelengths regardless of the direction of the pump sweep, this leads to an asymmetric transmission scan that is different depending on the sweeping direction [146]. In practice, this also means that only a blue-detuned pump can be thermally stable. A small increase in the pump wavelength will cause the resonance location to shift away from the pump in the red-detuned case at which point the power level in the resonator decreases and causes the resonance to shift even further. Paper E as well as Ref. [146] show this in action.

While this seemingly makes any state requiring \( \delta_0 > 0 \) impractical to achieve, it is worth noting that the Kerr effect also shifts the resonance in the same direction [145]. As seen in figure 3.5(a), the upper bi-stable branch has \( \delta_0 > 0 \) even though it is blue-detuned from the resonance peak. The net pump detuning will therefore correspond to the sum of the Kerr phase shift and the unloaded resonance detuning:

\[
\delta_{\text{net}} = \delta_0 - \phi_{\text{Kerr}} \\
= \delta_0 - \gamma PL.
\]

As the intracavity power, \( P \), close to resonance can reach several tens of Watts (in proportion to \( Q \) as described in Section 3.1.5), CW solutions can remain stable for small positive detunings. For solutions involving pulse forms beyond the CW state, a basic net detuning analysis can be made by looking at the phase shift accumulated by the CW pump during one roundtrip [28].
3. CHAPTER. MICRORESONATOR DYNAMICS
Chapter 4

Basic soliton dynamics in microresonators

In the previous chapter, we discussed the availability of several CW steady-state solutions to the Lugiato-Lefever equation. From these solutions, given that certain boundary conditions are met (described in equation 3.44), MI-like parametric gain can be achieved. While MI gives a hint at what comb initialization might look like, there is a richer dynamics available in nonlinear microresonators allowing several possible stable comb states. These include Turing patterns and circulating bright cavity solitons and dark pulses [147]. This chapter aims at discussing these stable comb states and their properties, particularly with respect to the metrics presented in Chapter 2.

4.1 Stable comb states

Perhaps the most intuitive comb states worth mentioning are those where the MI growth has cascaded into multiple comb lines and stabilized. These states go by the names of coherent MI combs or Turing patterns [147–150] after Alan Turing’s work on pattern formation in nature [151]. Turing pattern-based comb states have been shown to be stable and robust towards external perturbations [26, 150]. Figure 4.1 shows an example case of a stable MI-based comb with a 37 FSR line spacing matching the locations predicted in the previous section by equation 3.44. Nevertheless, the limited line numbers (as seen in Paper A and Refs. [26, 152]) have constrained their use in coherent communications experiments.
4.2 Bright solitons

In the anomalous dispersion regime, Turing pattern-based combs can be turned into bright cavity soliton states [87, 89, 148, 153]. They can typically be generated by careful tuning of the CW pump into the resonance. When tuning in a straight-forward manner (i.e. using linear ramping of the pump power or wavelength), the comb state has to go through a chaotic region however. The final state will thereby greatly depend on the initial noise conditions [121, 148, 153, 154]. It has however been shown that by taking a more advanced route in terms of ramping the pump power and detuning, this chaotic regime can be avoided [155]. In practice, reaching a stable single-soliton state requires feedback from the experimental setup [89, 156, 157]. Recently, several groups have shown that using careful tuning methods, this can allow for deterministic single-soliton generation by tuning the laser wavelength [158, 159] or the chip temperature [160].

The final steady-state can be described as the sum of a low-power CW wave (corresponding to the lower branch of the CW steady-state solutions) and a soliton pulse. For a lossless resonator system, the soliton pulse can be derived analytically from the Lugiato-Lefever equation [89, 148, 161, 162]:

$$A_{cs}(t) = \sqrt{\frac{2\delta_0}{\gamma L}} \text{sech} \left( t \sqrt{\frac{2\delta_0}{|\beta_2|L}} \right). \quad (4.1)$$

The above equation corresponds to a soliton with the following total energy,
4.2. BRIGHT SOLITONS

\( E_{cs} \), and pulse duration (at FWHM), \( \Delta t_{cs,3dB} \):

\[
E_{cs} = \int_{-\infty}^{\infty} |A_{cs}(t)|^2 dt = \frac{2}{\gamma} \sqrt{\frac{2\delta_0 |\beta_2|}{L}}, \tag{4.2}
\]

\[
\Delta t_{cs,3dB} = \sqrt{\frac{2|\beta_2|L}{\delta_0}} \log(1 + \sqrt{2}). \tag{4.3}
\]

This detuning-dependence of the soliton energy allows stabilizing the comb state without direct measurement of the detuning parameter [159]. In the above equations, the detuning looks like a free parameter. It also seems like maximizing it is of interest to achieve narrower (i.e. higher bandwidth) and stronger (i.e. higher conversion efficiency) solitons. In a real system with losses from absorption and outcoupling however, boundary conditions exist for it. The upper limit permitting stable bright solitons has been shown to be [137, 148, 162]:

\[
\delta_0 \leq \frac{\pi^2}{8} \frac{\gamma L \theta P_{in}}{(\alpha L + \theta)^2/4}. \tag{4.4}
\]

In the following equations and text, we will assume that we are always interested in the maximum allowed detuning. We can therefore replace the detuning parameter with the right-hand side of inequality 4.4. By comparing the outcoupled soliton energy, \( \theta E_{cs} \), with the input pump power over one roundtrip time (\( P_{in}/\text{FSR} \)), the maximum possible conversion efficiency, \( \eta \), can be calculated [128]:

\[
\eta = \frac{2\pi \text{FSR}}{\alpha L + \theta} \sqrt{\frac{|\beta_2| \theta^3}{\gamma P_{in}}}. \tag{4.5}
\]

To calculate the intra-cavity frequency spectrum of a soliton pulse, equation 4.1 has to be Fourier-transformed. When considering a pulse train of solitons, the envelope of the power spectral density also has to be multiplied by the repetition rate squared, yielding:

\[
P_{\text{env.}}(\omega) = 2\pi \frac{\pi |\beta_2|}{2\gamma} \text{sech}^2 \left( \frac{\omega}{\pi} \sqrt{\frac{L |\beta_2|}{2\delta_0}} \right) \text{FSR}^2. \tag{4.6}
\]

By again setting the maximum allowed detuning according to inequality 4.4, we can calculate the pump-power dependent spectral envelope. Finally, to see
4. CHAPTER. BASIC SOLITON DYNAMICS IN MICRORESONATORS

Figure 4.2. Simulated 50 GHz bright soliton comb showing the (a) intracavity time-domain picture and the (b) outcoupled spectrum. The red curve shows the expected theoretical envelope from equation 4.7. The comb parameters are as follows: $\beta_2 = -200 \text{ps}^2/\text{km}$, $\theta = 0.02$, $\alpha = 0.1 \text{dB/cm}$, $n_g = 2$, $\gamma = 2 \text{W}^{-1}\text{m}^{-1}$, $P_{in} = 20 \text{dBm}$, $\delta_0 = 0.077 \text{rad}$.

The outcoupled spectral powers, the coupling parameter, $\theta$, has to be included:

$$P_{\text{env, out}}(\omega) = 2\pi \frac{\beta_2}{2\gamma} \text{sech}^2 \left( \frac{\omega}{2\sqrt{\frac{\beta_2}{\gamma \theta P_{in}}} \text{FSR}^2 \theta} \right)$$

(4.7)

As shown in figure 4.2(b), this equation provides a good fit to simulated comb states. From this, one can further calculate the soliton’s 3 dB bandwidth in the frequency domain as well as the number of lines that fall within that bandwidth [128]:

$$\Delta f_{3\text{dB}} = \frac{2 \log(1 + \sqrt{2})}{\pi (\alpha L + \theta)} \sqrt{\frac{\gamma \theta P_{in}}{|\beta_2|}}$$

(4.8)

$$N_{3\text{dB}} \approx \frac{\Delta f_{3\text{dB}}}{\text{FSR}}$$

(4.9)

From a comb use case perspective a few trends and trade-offs are worth highlighting [128, 148, 163]:

- An increased pump power increases the comb bandwidth (equations 4.7 and 4.8) while decreasing the power conversion efficiency (equation 4.5).
- Doubling the comb FSR quadruples the comb lines’ powers (equations 4.6 and 4.7).
- The power conversion efficiency and number of generated comb lines are inversely related according to $\eta N_{3\text{dB}} = 4 \log(1 + \sqrt{2}) \theta^2/(\alpha L + \theta)^2$ (equations 4.5 and 4.9).
4.2. BRIGHT SOLITONS

Figure 4.3. Simulated 50 GHz bright soliton comb containing two circulating pulses showing the (a) intracavity time-domain picture and the (b) outcoupled spectrum. The red curve shows the expected theoretical envelope from equation 4.7. The comb parameters are as follows: $\beta_2 = -200 \text{ps}^2/\text{km}$, $\theta = 0.02\text{, } \alpha = 0.1 \text{dB/cm}$, $n_g = 2$, $\gamma = 2 W^{-1}\text{m}^{-1}$, $P_{\text{in}} = 20 \text{dBm}$, $\delta_0 = 0.077 \text{rad}$.

In other words, there exists a fundamental trade-off between the number of lines, the pump power, and the outcoupled line powers. When it comes to applications, these bright soliton states have been shown to be capable of producing a large number of comb lines spanning wide bandwidths. This has enabled impressive data communications demonstrations spanning both optical C and L-bands providing aggregate data rates of several tens of terabits/s [27]. Self-referencing is enabled by combs spanning close to, or even surpassing an octave [135, 164]. This has enabled the prospect of an integrated optical-frequency synthesizer [165]. Other recent developments include dual-soliton systems enabling chip-based dual-comb spectroscopy [73] and fast sub-micrometer distance measurements [166, 167]. In the quoted demonstrations (and others), bright soliton combs have proven a level of maturity and stability allowing research to be conducted with them as black-box coherent multi-wavelength light sources.

As a note of curiosity: The single-soliton case is not the only stable bright soliton state available. After moving through the chaotic region during the initialization process, the system can converge to a state containing multiple circulating bright solitons [158, 159, 168]. Such a state is shown in figure 4.3. In this situation, the power conversion efficiency increases linearly with the number of solitons in the cavity! The resulting spectral envelope contains “holes” however prohibiting applications that require all lines to have high powers. These multi-soliton states have been proposed to be useful as optical buffers [169], however they constitute a complete research topic in themselves [170].
4.3 Dark pulses

Recently it has been shown that a wide-bandwidth comb state can also be generated in the normal dispersion regime [91, 171]. One interesting aspect of these states is that the power conversion efficiency can reach $\eta > 30\%$ even with more than $\sim 40$ lines present [28, 172]. This is in contrast to similar bright soliton systems where $\eta$ is typically limited to the order of a few percent. Whereas the bright solitons correspond to circulating bright pulses inside the resonator cavity, the broad-band normal-dispersion states can contain circulating dark pulses. While these states are in some ways analogous to the dark soliton solution in fibers, they have to comply with the periodic boundary condition imposed by the resonator. This prohibits the $\pi$ phase change of the classical dark fiber solitons. Microresonator dark pulses have however successfully been described and simulated in the form of “platicons” [173, 174], or switching waves [175–179]. In contrast to the intracavity field for the bright cavity solitons that consists of the low-powered CW state and a pulse, the switching wave description approximates the dark pulses to switch between the low-powered CW state and the high-powered CW state. This difference is shown in figure 4.4. The features yielding a comb in the spectral domain can therefore be attributed to the transition between the two CW states. Similarly to the bright soliton combs, it is possible to stabilize a dark pulse in several steady states corresponding to a different order circulating dark pulse. Figures 4.5 and 4.6 show two such example cases with the bright
4.3. DARK PULSES

Figure 4.5. Simulated 50 GHz dark pulse comb showing the (a) intracavity time-domain picture and the (b) outcoupled spectrum. The red curve shows the expected theoretical envelope for bright solitons from equation 4.7. The comb parameters are as follows: $\beta_2 = 200 \, \text{ps}^2/\text{km}$, $\theta = 0.02$, $\alpha = 0.1 \, \text{dB/cm}$, $n_g = 2$, $\gamma = 2 \, \text{W}^{-1}\text{m}^{-1}$, $P_{\text{in}} = 20 \, \text{dBm}$, $\delta_0 = 0.0552 \, \text{rad}$.

soliton envelope overlaid. An important difference with regards to bright cavity solitons is that in the dark pulse state, the lobes resulting from entering a higher-order state do not result in weaker components in the spectrum. On the contrary, they help with broadening (and flattening!) the output comb spectrum.

An analytical analysis comparable to the bright solitons cannot be performed without a closed-form expression. Numerical analysis however (as will be described in Paper S) shows that the trends are very similar. In the same manner, increasing the comb bandwidth results in weaker comb lines overall. The design of a comb source will therefore be very much application-dependent. For communications, only comb lines within a set bandwidth (such as the C-band) are typically of interest. In Paper D, 20 comb lines within the C-band are generated using a dark pulse comb source pumped with less than 0.5 W off-chip power. Owing to the high (> 20%) power conversion efficiency of the device, these lines allow data to be encoded using modern high-order coherent modulation formats.

As discussed in the previous chapter, the Lugiato-Lefever model predicts that for CW pumps (avoiding significant red detuning), a resonator needs anomalous dispersion to initialize a FWM process. Dark pulse combs however operate in the normal dispersion regime. To simulate dark pulse comb states, we therefore typically initialize the system with a square wave according to the levels of the bistability curve (as seen in figure 4.4 (a)) [91]. In this manner, the system converges to a dark pulse state without any ramping of the pump parameters. In a real-world system however, we do not have
Figure 4.6. Simulated 50 GHz dark pulse comb containing a higher-order dark pulse showing the (a) intracavity time-domain picture and the (b) outcoupled spectrum. The red curve shows the expected theoretical envelope for bright solitons from equation 4.7. The comb parameters are as follows: $\beta_2 = 200 \text{ps}^2/\text{km}$, $\theta = 0.02$, $\alpha = 0.1 \text{dB/cm}$, $n_g = 2$, $\gamma = 2 \text{W}^{-1}\text{m}^{-1}$, $P_{\text{in}} = 20 \text{dBm}$, $\delta_0 = 0.0552 \text{rad}$.

The luxury of picking the starting conditions. Most demonstrated systems (including Refs. [91, 141] and the combs used in Papers A-D) are therefore enabled by multi-mode dynamics described in the previous chapter. As these systems are not tunable after fabrication, the comb’s central wavelength is defined by the location of the modal coupling. By adding a second resonator to the system, it is possible to remove the multi-mode requirement for the comb generation [142, 180]. By thermally tuning the two resonators, their respective modes can couple in a tunable manner allowing post-fabrication control over the resulting comb’s central wavelength. This has the potential to simplify and expand the possible scenarios where dark pulse combs can be useful.
Chapter 5

Future outlook

The results presented in this thesis provide a starting point for employing dark-pulse combs in coherent transmission applications. There are several paths along which one can continue however. Here, I will highlight two areas of personal interest:

- **Exploit comb coherence in transmission experiments.**
  As demonstrated in several publications including Refs. [25-27] and Papers A and D, microresonator combs are already today used in laboratory-based coherent communications experiments. These experiments however rely on the combs merely being a replacement for a set of lasers. While there are power efficiency gains to be had by using integrated comb sources (see for example the calculations in the supplementary material of Ref. [27]), they have the potential to yield further advantages. Experimentally proving that microresonator combs can allow for joint processing of several data channels (in terms of phase tracking, as with electro-optic combs in Ref. [105] or multichannel digital back-propagation as in Ref. [98]) as well as allowing dense channel spacing (as with electro-optic combs in Ref. [104]) would further increase their attractiveness. Enabling joint processing experiments will require using identical combs in the transmitter and receiver (as has been proven possible in Ref. [27]) together with a clever multi-channel transmitter and receiver setup. To enable demonstrations with ultra-dense channel spacing, microresonator combs are needed with line spacings on the order of 50 GHz. While manufacturing dense combs is challenging, recent demonstrations have included both bright solitons [181] and dark pulses [172].
Explore and optimize the operation of dark pulse combs.

One of the differences between the bright soliton and dark pulse comb states is their pump laser detuning. Whereas bright soliton states have been estimated and measured to operate in a net red-detuned manner [158, 182], dark pulse states are expected to stay on the blue side [28]. While the implications of this are not entirely clear, it is nevertheless a parameter that should be measured and verified experimentally.

In Paper B we observed a small, yet noticeable, relative phase noise increase on microresonator comb lines located far away from the pump. This intriguing trend has at present no explanation and requires further analysis on combs of various types as it will likely have an effect on the performance of joint phase tracking algorithms.

Finally, the optimized dark pulse comb states found in simulations (as discussed in the supplementary materials to Paper D and in what will be presented in Paper S) will require experimental verification. Key trends, including relationships between conversion efficiencies, comb line numbers and pump power requirements will have to be tested. Having access to these relations will be critical in designing optimal comb sources for optical communication systems of any bandwidth.
Chapter 6

Summary of papers

Paper A


In this paper we present the results from two coherent long-haul transmission experiments. We transmitted PM-QPSK-modulated data over more than 6000 km and PM-16QAM-modulated data over more than 700 km using a recirculating fiber loop. The light source consisted of seven (resp. six) lines generated using a low-noise microresonator-based frequency comb manufactured at Purdue University, USA. This is the first demonstration of long-haul data transmission using microresonator-based combs as light sources showing that the technology fits the requirements of long-haul data transmission. To describe the initialization mechanisms for the used combs, the paper also includes characterization of the mode coupling present in the resonator used for the second transmission experiment.

My contribution: I performed the transmission measurements with support from the co-authors, implemented the DSP code, characterized the mode coupling in the system, wrote the paper with support from the co-authors, and presented part of the results at CLEO in San Jose, USA in 2016.
Paper B


In this work, we perform initial characterizations of the frequency noise present in a normal dispersion microresonator comb. The microresonator was the same comb as the one used in the 16QAM transmission experiment in Paper A. We conclude that, while mostly depending on the pump laser, there are scaling effects present. While the scaling is small enough not to be noticed in the 16QAM experiment in Paper A, it is measurable.

My contribution: I performed the device characterization and data processing. I also wrote the paper with support from the co-authors and presented the work at OFC in Los Angeles, USA in 2017.

Paper C


In this work we set up a basic stabilization scheme for a dark-pulse microresonator comb. The microresonator was supplied by our collaborators at Purdue University, USA. By continuously measuring the outcoupled power and automatically adjusting the pump frequency when it was drifting, we could keep the total power in the comb stable over several hours.

My contribution: I performed the measurements and wrote the paper with support from the co-authors. I presented the work at CLEO Europe in Munich, Germany in 2017.
Paper D

“High-order coherent communications using mode-locked dark pulse Kerr combs from microresonators”, *Under review.*

In this paper we present the results from a 20-channel coherent transmission experiment. We modulated data using PM-64QAM onto the lines of a dark-pulse microresonator comb fabricated at Purdue University. The results constitute the highest-order modulation formats encoded onto any integrated comb source to date. We furthermore kept the pump laser power at a level compatible with state-of-the-art integrated lasers. Together with included simulations using a more optimized comb, we claim that dark-pulse microresonator combs can at the same time be compatible with integrated power levels and high bit rate communications.

**My contribution:** I performed the transmission measurements together with the co-authors, implemented the DSP code, characterized the dark pulse state and the comb line spacing stability, and wrote the paper with support from the co-authors.

Paper E


Here we measured the changes in four-wave mixing efficiency inside a triply-pumped normal dispersion microresonator while varying the relative phases of the pump waves. The three waves were generated using a tunable laser and electro-optic modulator and were set to wavelengths matching three adjacent resonances in the microresonator. The microresonator was manufactured in a multi-project wafer run by LioniX in the Netherlands. Additionally a simplified analytical model was developed that qualitatively matches the measurements while more complete numerical simulations were performed that also match quantitatively. Simulations using similar pump parameters but with an anomalous dispersion resonator were also performed. The results of experiments, simulations and analytical model all indicate that controlling the relative phases in the pump waves is of critical importance when maximizing the four-wave mixing efficiency.
My contribution: I performed the measurements, calculated the model with support from the co-authors, implemented the simulations, and wrote the paper with support from the co-authors. I also presented the work at CLEO Europe in Munich, Germany in 2015.
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6. CHAPTER. SUMMARY OF PAPERS


Appendices

Appendix A — Offline coherent receiver DSP

Digital signal processing is required to recover the transmitted data from a received signal at the end of an optical fiber link. The flowchart in figure 6.1 describes the code of the DSP used in Paper D. The signal recorded in a real-time oscilloscope needs to be processed in several steps:

1. Impairments originating from the receiver need to be handled first. This starts by compensating for relative delays caused by varying electrical lengths of cables and waveguides. Second, any remaining phase offsets and power differences in the varying arms need to be compensated for. This can be done using Gram-Schmidt orthogonalization [183]. Finally, the received signal is typically resampled to an integer number samples per symbol, two in the case of Paper D.

2. Static link effects known in advance can now be handled. In our case this meant compensating for the fiber’s chromatic dispersion.

3. Dynamic link (and receiver) effects need to be handled by a multi-tap adaptive equalizer. Figure 6.2 (a) describes its basic operation. By looping over the signal, an equalizers taps can dynamically handle polarization-mode dispersion as well as residual chromatic dispersion. To train the equalizer taps in the laboratory scenario, they were looped over the signal several times. This section of the DSP also includes frequency offset estimation between the local oscillator and the transmitter laser [184] as well as tracking of the carrier’s phase noise as shown in figure 6.2 (b) [185].

4. The DSP now handles the remaining impairments from the transmitter. In practice, we perform a second Gram-Schmidt orthogonalization.
Figure 6.1. Flow chart showing the main components of a DSP algorithm for decoding $M$-ary QAM.

\[ A_I = \mathcal{F}^{-1} \{ \exp(i\omega \Delta t_I) \mathcal{F} \{ A_I \} \} \]
\[ A_Q = \mathcal{F}^{-1} \{ \exp(i\omega \Delta t_Q) \mathcal{F} \{ A_Q \} \} \]

Gram-Schmidt orthogonalization [183]

\[ A = \mathcal{F}^{-1} \left\{ \exp(iL \frac{\beta_2}{2} \Delta \omega^2) \mathcal{F} \{ A \} \right\} \]

Tap convergence and FFT-based frequency offset estimation [184]

Gram-Schmidt orthogonalization [183]
Figure 6.2. (a) The adaptive equalizer transverses over the signal. For each symbol it calculates an error metric and adjusts its taps to minimize it. In the CMA equalizer case, the error metric is the difference between the symbol power and a prescribed target reference power. In the DDLMS case, the error metric is the distance between the decoded symbol and the nearest available constellation point. (b) In the DDLMS case, the phase of the signal has to be tracked before the error can be estimated. Blind phase tracking involves testing a set of phases and assuming the one that gives the lowest mean square error over a block length [185].

5. Finally, the symbols extracted from the final equalizer loop are decoded to bits (typically using Grey coding). In the laboratory scenario we now compare the results to the known transmitted bit sequence yielding a final system bit error ratio.

Appendix B — Microresonator simulation

The simulations in Paper D were performed using the Ikeda map approach to allow coupling in random noise after each roundtrip as well as during propagation [120–122]. To reproducibly converge to a dark pulse comb, the intracavity field was initialized with a square wave according to [91]. Figure 6.3 (a) contains a flow chart of the method. The propagation along the resonator was done using the standard split-step technique [33] with figure 6.3 (b) showing the steps.
6. CHAPTER. SUMMARY OF PAPERS

(a) Main simulation loop

Initialize system
- Generate pump field
- Generate resonator intracavity field

Loop n times
- Directional coupler
- Perform waveguide propagation

Main simulation

$A_{\text{pump}} = \mathcal{F}^{-1}\left\{ \sqrt{P_{\text{in}}} + \sqrt{\frac{\hbar \omega}{2}} \exp(i \phi_{\text{rand}}) \right\}$

Start $A_{\text{in}}$ with a square wave with 50% duty cycle. The top and bottom levels are given by the bistability curves [91].

Couple light in and out of the resonator:

$A_{\text{out}} = A_{\text{pump}} \sqrt{1 - \theta} - A_{\text{in}} \exp(i \phi_0) \sqrt{\theta}$

$A_{\text{in}} = A_{\text{in}} \exp(i \phi_0) \sqrt{1 - \theta} + A_{\text{pump}} \sqrt{\theta}$

(b) Waveguide propagation

Loop m-1 times
- Perform half nonlinear step
- Perform linear step
- Perform nonlinear step
- Perform linear step
- Perform half nonlinear step

Perform linear step:

$A_{\text{in}} = \mathcal{F}^{-1}\left\{ \exp(i \beta \Delta L) \exp\left(-\frac{\alpha L}{2}\right) \mathcal{F}\{A_{\text{in}}\} \right.$

$+ \sqrt{1 - \exp(-\alpha \Delta L)} \sqrt{\hbar \omega/2} \exp(i \phi_{\text{rand}}) \}$

Perform nonlinear step:

$A_{\text{in}} = A_{\text{in}} \exp\left(i |A_{\text{in}}|^2 \gamma \Delta L \right)$

Figure 6.3. (a) Flow chart showing the main steps in the Ikeda map simulation. The main simulation loop typically runs $n > 2^{14}$ roundtrips until the system converges. (b) The waveguide propagation is performed according to the standard split-step method [33]. To keep noise levels realistic also in the presence of high losses, each loss step (in the linear part) also couples in quantum noise at the appropriate level.

Appendix C — Microresonator stage

To enable pumping and stabilizing the microresonator comb operation, the stage shown in figure 6.4 was used. A continuously tunable pump laser (Toptica CTL 1550) and a high-power amplifier was used to generate the pumping
laser light. Using an optical bandpass filter centered around the pumping wavelength of 1540 nm (in practice a WDM-coupler selecting ITU grid channel 47), we could minimize the amount of excess ASE noise entering the resonator. The chip itself was placed on a piezo-controlled positioning stage. The stage contains a vacuum pump to minimize movement of the chip over long timescales as well as a temperature controller stabilizing the stage temperature. See figure 6.5 for a labelled photograph. Following the chip, an OSA (Ando AQ6317B) was used to monitor the optical spectrum. To allow locking of the comb state, one of the newly generated comb lines was additionally filtered out and measured in a photodiode. The low-frequency ($< 10$ kHz) component of the power drift was fed back to the laser providing a locking point. Any mid and high-frequency noise measured by the photodiode was measured in an electrical ESA (Rigol DSA815) allowing visual verification of the low-noise comb state.
Figure 6.4. Microresonator stage setup with all the required components.
Figure 6.5. Labelled photo of the microresonator mounting stage.
Papers
Long-haul coherent communications using microresonator frequency combs


Long-haul coherent communications using microresonator-based frequency combs

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Abstract: Microresonator-based frequency combs are strong contenders as light sources for wavelength-division multiplexing (WDM). Recent experiments have shown the potential of microresonator combs for replacing a multitude of WDM lasers with a single laser-pumped device. Previous demonstrations have however focused on short-distance few-span links reaching an impressive throughput at the expense of transmission distance. Here we report the first long-haul coherent communication demonstration using a microresonator-based comb source. We modulated polarization multiplexed (PM) quadrature phase-shift keying-data onto the comb lines allowing transmission over more than 6300 km in a single-mode fiber. In a second experiment, we reached beyond 700 km with the PM 16 quadrature amplitude modulation format. To the best of our knowledge, these results represent the longest fiber transmission ever achieved using an integrated comb source.

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OCIS codes: (060.1660) Coherent communications; (190.4390) Nonlinear optics, integrated optics.

References and links


1. Introduction

The advent of self-referenced femtosecond mode-locked lasers [1,2] allowed for establishing a coherent link between radio frequency and optical signals obtained from lasers or atomic transitions. This technology is finding an increasing number of applications in precision frequency synthesis and metrology [3]. Microresonator-based frequency combs have been identified as a key technology to reach a similar level of precision in a monolithic platform. This technology provides an opportunity to attain line spacings significantly higher than what can be achieved with standard mode-locked lasers [4,5]. In addition, high-Q microresonators can be fabricated using standard semiconductor fabrication processes, see e.g [6–11]. As a result, microresonator frequency combs are opening up a whole new range of technological possibilities. Recent demonstrations include self-referencing [12,13], optical clocks [14], radio-frequency photonics [15,16], spectroscopy [17,18], optical waveform synthesis [19] and high-capacity communications [20].

In fiber-optic communications, an integrated multi-wavelength light source would provide the fundamental carriers on which to encode data using e.g. wavelength-division multiplexing (WDM). A frequency comb would allow replacing a large number of WDM channel light sources with a single laser. To date this has been explored principally with combs generated via periodic electro-optic modulation of a continuous-wave input laser, often supplemented with nonlinear spectral broadening in an optical fiber [21–25]. These electro-optic combs have been used as communications light sources both for short-distance on-off keying systems [21] as well as in long-haul demonstrations with all-optical orthogonal frequency-
division multiplexing [26] and coherent [27] modulation formats. Absolute frequency accuracy is in principle not needed, but line spacing stability provides an opportunity to efficiently mitigate inter-channel nonlinear distortion via nonlinear pre-compensation [28]. Since a self-referenced light source is not necessary, there is a multitude of integrated comb sources that can be considered, such as integrated electro-optic combs [29], silicon-organic hybrid modulators [30], passively mode-locked lasers [31] or quantum-dash mode-locked lasers [32]. The rationale for using microresonator-based frequency combs lies in the fact that they can be generated using high-performance silicon nitride high-Q microresonators [6,8,11,33]. This technology is compatible with standard CMOS fabrication processes and recent demonstrations have shown that it is possible to realize multi-layer integration with active silicon components [34]. This significant achievement opens up the possibility to in the future realize a fully integrated comb-based transceiver with silicon photonics technology. Indeed, silicon nitride microresonator combs have shown a level of performance compatible with coherent communications [20]. Bright temporal solitons in silicon nitride microresonator combs have led to impressive demonstrations attaining 50 Tb/s aggregate data rates [35]. These results demonstrate the potential to achieve hundreds of lines in a single device by proper dispersion engineering. Demonstrations have however so far not been performed for long-haul distances beyond a few fiber spans.

In this work we demonstrate the results from two long-haul communication experiments using microresonator combs designed in silicon nitride waveguides. Working in the stable modulation instability regime [36,37], they provide a handful of high-powered lines enabling long-distance transmission. We modulated data using polarization-multiplexed quadrature-phase shift keying (PM-QPSK) and 16-quadrature amplitude modulation (PM-16QAM) on two different comb devices. We achieved propagation distances of 6300 km using 12.5 GBd PM-QPSK and 700 km with 20 GBd PM-16QAM. These represent the longest transmission distances ever achieved using an integrated comb source. This work extends results presented in [38] and [39] by increasing the data rate using PM-16QAM as well as characterizing the comb generation process.

2. Comb generation

The devices consisted of rings with 100 µm radius leading to a comb line spacing of multiples of 230 GHz. SiN microresonator combs with smaller spacings are possible [35,40]. Practical challenges that need to be addressed when going for longer cavity lengths include maintaining low waveguide losses as well as limiting the effects of modal crossings in the ring waveguide. The latter effects could be addressed e.g. by using an arrangement of linearly coupled single-mode cavities [41]. The devices used in this work consisted of a single microring equipped with a drop port as shown in Fig. 1(a). The drop port assisted in filtering out amplified spontaneous emission (ASE) noise that was left around the pump laser while at the same time limiting the amount of pump light that was present at the output port [42]. Contrary to the cavity soliton-based microresonator combs that require anomalous dispersion waveguides, our microresonators were designed to operate in the normal dispersion regime. To initialize the combs from a single continuous wave (CW) laser source using degenerate four-wave mixing, it is however still required to have locally anomalous dispersion. This can be achieved through local perturbations. In our case, the perturbation was caused by coupling between transverse modes [43]. Figures 1(b)-1(d) illustrate the effect of modal coupling where in certain frequency regions the resonance locations, and thus the effective index, are strongly perturbed by the presence of a second (higher order) mode. This leads to anomalous dispersion locally, enabling the degenerate four-wave mixing process [43].
Fig. 1. (a) Microscope image of one of the microresonators used in this work. (b) Transmission scan of a multimode device highlighting a region where two modes couple linearly. By dividing the spectrum into regions spaced with 1 FSR, we can illustrate the coupling effect in subfigures (c) and (d). As the resonance locations move closer, they will repel each other if there is coupling between them, leading to the resonances being slightly offset in the shaded region. Since this is locally changing the effective index of the waveguide, the dispersion of the device will also be greatly affected. A more detailed description of the effects of modal coupling is given in [43].

The comb devices were excited in the soft-excitation regime using thermal wavelength tuning of an external cavity pump laser with a sub-100 kHz specified linewidth. The pump laser was amplified with a high-power erbium-doped fiber amplifier (EDFA) after which a 1 nm wide optical band pass filter was placed to remove excess ASE noise. The EDFA output power was set to 27 dBm in the first experiment (corresponding to ~24 dBm on-chip power) and 30 dBm in experiment two (corresponding to ~27 dBm on-chip power). After the pump EDFA, the chip was placed on a temperature controlled stage constraining any temperature variations to less than 0.01 °C. The chip was coupled to and from using lensed fibers placed into U-grooves etched into the device [42] permitting stable operation over several hours. A resonance several nanometers away from the mode crossing was pumped. The comb state is coherent and it operates in the stable modulation instability region [37]. Figures 2(a) and 2(c) show the optical spectra of the combs used in our experiments. The comb in Fig. 2(c) was generated from the ring with the transmission scan shown in Fig. 1(b). Two different resonators were used for the two experiments due to damage sustained to the first device. The waveguides of both microresonators had the same thickness at 600 nm, while their widths were 3 µm and 2 µm respectively. The excited mode was the first-order X-polarized mode and had a dispersion of roughly 250 ps²/km [42]. The RF spectra in Figs. 2(b) and 2(d) indicate that the combs were operating in a low-noise state [44]. The total conversion efficiencies (adding the through and the drop port) of both used combs were measured to be above 10% (see Appendix).
3. Data transmission and results

Before modulating data onto the comb lines, we also placed a flattening stage consisting of a programmable pulse shaper and an EDFA ensuring that all lines had equal power. This resulted in a flat spectrum with an optical signal-to-noise ratio (OSNR) of >35 dB per line at 0.1 nm resolution before data modulation. To emulate a WDM system in the laboratory without the large number of components required, all the lines of the frequency comb were instead modulated simultaneously using the same transmitter. See Fig. 3(a) for a description of all the necessary components.

**The transmitter**

The transmitter consisted of an electro-optic IQ-modulator modulating all the comb lines simultaneously. The modulator was followed by a dual-polarization emulation stage using a polarization maintaining fiber (PMF) corresponding to a delay of more than 20 symbols. The intrinsic dispersion of a 27 km single-mode fiber (SMF) was used to delay and decorrelate the different channels with respect to each other, corresponding to a delay of more than 10 symbols. Since our carriers were spaced several hundred GHz apart, linear crosstalk between the channels is intrinsically not expected to be a problem, thereby we can avoid the widely used even-odd modulation scheme that would require an extra transmitter. Using fiber dispersion as means to decorrelate the channels allows us to emulate a scenario where different data were encoded on each channel to ensure that any correlated nonlinear crosstalk is also avoided. The launch power into this section was only $-10 \text{ dBm}$ to avoid nonlinearities in this span. Different data sources were connected to the modulator in the two experiments. For the longest reach one, a 12.5 Gbd QPSK signal was generated using a pseudorandom binary sequence pattern generator with a pattern length of $2^{15}-1$ bits (PRBS-15). For the second measurement, the data source was replaced with a newly acquired arbitrary waveform generator (AWG) allowing for the generation of a more spectrally dense QAM signal. The 20Gbd 16QAM signal consisted of a triply oversampled PRBS-15 bit pattern with an added
trailing zero to comply with the AWG’s pattern length requirements. As previously mentioned, two different microresonators were used in the two experiments. In the QPSK case, a single free spectral range (FSR) spaced comb (corresponding to 230 GHz line spacing) with 7 lines covering a part of the C-band (see the dashed box in Fig. 2(a)) was used. In the 16QAM system a second microresonator giving a 3 FSR spaced comb (corresponding to 690 GHz line spacing), covering the full C-band with 6 lines (see Fig. 2(c)) was used instead.

Both comb and transmitter combinations were tested in a back-to-back noise loading setup, where progressively more ASE noise was added to the signal after modulation. The OSNR at which a bit error rate (BER) of $10^{-3}$ was received was then recorded for each comb line. Reference measurements were done where free-running lasers were instead tuned to the wavelength locations matching the comb lines. The reference lasers were external cavity lasers with $<100$ kHz specified linewidth, nominally identical to the one used as pump for the microresonator. This was done to study any implementation penalty caused by the combs themselves. Figure 3(b) shows the results for the two systems. The conclusion is that the comb lines do not cause any significant penalty. The permitted transmission distance will therefore be limited by the comb lines’ initial OSNR before modulation and the imperfections in the various other pieces of equipment that are part of the setup. While we have observed a variation in instantaneous linewidth across the bandwidth in our microresonator combs [39], such a degradation is not expected to cause significant penalties in transmission given the requirements for QPSK and 16QAM [45]. We are currently investigating this aspect.

Fig. 3. (a) Schematic of the experimental setup used for the comb generation and the data modulation. The PMF delay and polarization beam combiner (PBC) were used to emulate a dual polarization transmitter. (b) Results for back-to-back noise loading measurements of both systems, the QPSK transmitter had an implementation penalty of $1.0 \pm 0.2$ dB while the 16QAM transmitter had an implementation penalty of $1.9 \pm 0.4$ dB. The implementation penalties for the comb lines do not significantly differ from those for a free-running laser source.
The recirculating loop

A recirculating loop with two 80 km fiber spans inside was used to emulate the transmission link. Figure 4(a) shows a schematic of its components. The SMF spans were preceded by a flattening stage and a tunable attenuator ensuring that the launch power of each line in the comb was kept at optimum. For the QPSK case the optimum total launch power was found to be 4 dBm while for the shorter distance 16QAM experiment it was 6.5 dBm. The OSNR at the input of the first fiber span was above 29 dB (normalized to 0.1 nm resolution) for all channels in both systems. The pulse shapers used for flattening also attenuated out-of-band noise. This helped to ensure that the launched signal power was maintained after multiple roundtrips. It is however worth noting that the rightmost channel received both less gain as well as greater frequency dependence of the gain compared with the other channels, see Fig. 4(c). Each SMF span was followed by an EDFA (with noise figures around 5.2 dB) to compensate for the losses. Additionally, the loop also contained a polarization scrambler synchronized to the switches and the loop roundtrip time. This way, by iterative optimization of the flattening stages it was possible to transmit the signal over multiple roundtrips.

The receiver

In the final part of the setup, there was a standard coherent receiver stage. One comb line was filtered out at a time and combined with a tunable local oscillator. A coherent receiver containing a polarization-diverse optical hybrid was connected to four ports of a 50 GSamples/s, 23 GHz bandwidth real-time oscilloscope, see Fig. 4(c). Batches of 2 million samples were recorded and processed offline. First the receiver imbalance was compensated for, after which the data was resampled to twice the baud rate. The dispersion from the fiber link was then compensated for after which an adaptive equalizer and a frequency and phase estimator performed the final processing. For the QPSK signal, the constant modulus algorithm was used together with an FFT-based frequency offset estimator and a Viterbi-Viterbi phase estimator. In the 16QAM case, a decision-directed least mean square equalizer was used with an FFT-based frequency offset estimator and a blind phase search algorithm [45]. Finally, the BER was calculated by comparing the decoded bits with the known transmitted sequences. By using a recirculating loop, we had the ability to vary the transmission distance, allowing us to find the maximum permitted distance given the chosen...
pre-forward error correction (FEC) BER of $10^{-3}$ (allowing for FEC overheads below 6.25% [46]).

**Results**

The two chosen modulation formats and symbol rates permitted transmission over different distances (see Fig. 5). With the QPSK system, at a cumulative pre-FEC bit rate of 350 Gb/s, we reached more than 6300 km with all the lines, three of which reached beyond 8000 km, resulting in a capacity-distance product above 2 Pb/s/km, assuming 6.25% FEC overhead. 8000 km agrees well with expectations from Gaussian noise model-based calculations [47] taking into account the OSNR requirements from Fig. 3(b). Since the higher bit rate 16QAM system (at a cumulative pre-FEC bit rate of 960 Gb/s) requires a higher OSNR at the receiver [48], the permitted distance for the second experiment was instead 700 km, with three lines going beyond 950 km, yielding a capacity-distance product above 630 Tb/s/km. Figures 5(a) and 5(b) show how the BER varied as a function of transmitted distance for both systems. Owing to the large channel spacing, both systems showed net spectral efficiencies around 0.2 b/s/Hz. In both cases, the outermost line at 1566 nm suffered a penalty in terms of distance. This is in agreement with the degraded gain performance observed in Fig. 4(b) owing to the fact that the channel is outside the specifications of the loop EDFAs. We therefore conclude that the distance penalty is not due to the comb source itself.

![Fig. 5. (a) and (b) Result of the PM-QPSK and the PM-16QAM transmission experiments showing BER as a function of distance in the recirculating loop. The insets display received and decoded signal constellations with bit error rates below $10^{-3}$.](image)

**4. Summary**

In summary, we have demonstrated experimentally that long-haul transmission is possible using silicon nitride microresonator-based frequency comb technology. Using QPSK and 7 comb lines and 16QAM with 6 lines, we achieve what we believe are the longest demonstrated coherent transmission links with an integrated comb source. The low-complexity QPSK format permitted transmission over transatlantic distances. The higher symbol rate 16QAM also permitted long-haul communications albeit at shorter distances. In
the future, more WDM channels will be needed to fill the whole C band with a line spacing compatible with the ITU grid. We envision that the use of mode-locked dark pulses in normal dispersion combs [49], with recently predicted [50] and demonstrated conversion efficiencies >30% [51] will provide a viable solution to achieve a favorable scaling with the number of channels and attain high spectral efficiency. The results of this work widen the scope of the usability of silicon photonics-based combs from short-reach high-capacity links to long-haul links, thus further opening up the path to chip-based transceivers where the coherent nature of frequency combs can be taken advantage of.

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**Appendix – Comb conversion efficiency**

The most straight-forward way of defining conversion efficiency is the relation between the optical power in all the newly generated comb lines (excluding the remaining pump line power) compared to the power in the initial pump [51]. Both measurements should be done in the bus waveguide, adjacent to the ring:

\[ \eta_{\text{comb}} = \frac{\sum P_{\text{comb}}}{P_{\text{pump}}} \]  

![Fig. 6.](image) (a) Sketch of a microresonator displaying the measurable power locations (\(P_n\) and \(P_{\text{out}}\)), the internal locations where the power levels need to be calculated as well as the loss elements. (b) Recorded spectra from the through and the drop ports of the two used microresonators. The displayed conversion efficiencies were calculated by measuring the powers in all the lines except for the pump line and comparing it to the throughput power in the off-resonant pumping case. The powers were measured using a grating-based optical spectrum analyzer with 0.1 nm resolution. In both devices, the total conversion efficiency (adding the through and the drop port) exceeded 10%.
Since the optical powers inside the microresonator device cannot be measured directly, we have to extract the relevant information from measurements done at the output ports of our device. Figure 6(a) shows a schematic of the device, the locations where the optical spectrum should be measured and the different loss elements that are not part of the comb generation process itself. Since the device is coupled into using tapered fibers at both ends, we will incur coupling losses at both facets, $\alpha_{c1}$ and $\alpha_{c2}$. In addition there will also be some (possibly insignificant) losses due to absorption in the bus waveguide, $\alpha_{wg}$. In practice, the spectrum can only be measured at the input and the output. The total loss can be measured by pumping the ring resonator off resonance and measuring the output power at the through port:

$$P_{\text{out,off}} = P_{\text{pump}}\alpha_{wg} \alpha_{c2} = P_{\text{in}}\alpha_{c1}\alpha_{wg} \alpha_{c2}.$$ (2)

After shifting the laser into resonance and initiating the comb, the spectrum can again be measured at the through port giving us the on-resonance output powers:

$$P_{\text{out,on}} = P_{\text{out,pump}} + \sum P_{\text{out,combines}} = P_{\text{out,pump}} + \alpha_{wg} \alpha_{c2} \sum P_{\text{combines}},$$ (3)

where $P_{\text{out,pump}}$ is the power left in the pump line at the output. From Eqs. (1), (2), and (3), we can now extract the conversion efficiency:

$$\eta_{\text{comb}} = \frac{P_{\text{out,on}} - P_{\text{out,pump}}}{P_{\text{in}}\alpha_{c1}\alpha_{wg} \alpha_{c2}} = \frac{P_{\text{out,on}} - P_{\text{out,pump}}}{P_{\text{out,off}}}. $$ (4)

Using Eq. (4) we can thus measure the conversion efficiency of our comb by only looking at the through-port and taking optical spectra in on- and off-resonant pumping situations. A similar calculation can be done for the drop port, assuming the chip-to-fiber coupling losses are identical for the two ports giving effective conversion efficiencies for both output ports.

Figure 6(b) shows the values for the combs that were used in our experiments. The two devices had slightly different gap distances between the ring and the through ports (500 nm vs 300 nm) while having the same 500 nm gap to the drop port leading to the difference in the ratio between the through and drop port conversion efficiencies.
Frequency noise of a normal dispersion microresonator-based frequency comb


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Frequency Noise of a Normal Dispersion
Microresonator-based Frequency Comb

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Abstract: Using delayed self-heterodyne coherent detection, we characterized the FM noise across the C-band of a widely spaced microresonator-based frequency comb. The resulting linewidth depends on both the pump laser and the comb line position.

OCIS codes: (060.1660) Coherent communications; (190.4390) Nonlinear optics, integrated optics.

1. Introduction

Recent experiments have demonstrated that microresonator-based Kerr combs are promising multi-wavelength integrated light sources for use in wavelength-division multiplexed (WDM) coherent communication systems, both in high-capacity single-span [1] and long-haul systems [2]. Kerr combs implemented in silicon nitride high-Q microresonators [3] offer a viable path to co-integrate the technology platform with additional transmitter components [4].

In order to use higher-order modulation formats with the lines of the Kerr comb, a particular emphasis must be placed on their linewidth properties. In microresonator combs this will be ultimately dictated by the laser pump linewidth and the nonlinear comb dynamics [5]. Unlike laser arrays, chip-scale comb generators may show a distinct linewidth behavior for different frequency lines. However, this topic has not been thoroughly investigated for Kerr combs.

Here, we carry out an experimental study of the linewidth variation across a low-noise Kerr comb centered in the C-band. The comb operated in the coherent modulation instability regime [6]. We analyze the FM noise of the instantaneous frequency [7], which allows us to distinguish the dominant frequency noise contributions for different time scales. We find that in the region relevant for coherent optical communications, the linewidth increases slightly as it moves away from the pump, regardless of the laser pump source. Although the impact is not severe for 20Gbaud 16QAM modulation when using a high-quality laser pump, these results point out phase noise dynamics that may be relevant for higher-order constellations.

2. Motivation – frequency noise impact on coherent communications

In the following we describe an experiment that motivated us to look further into the linewidth properties of the comb. We used a Kerr comb implemented in a high-Q (~$10^9$) microresonator with 230GHz free spectral range (FSR). The fundamental mode displays normal dispersion and, owing to linear coupling to a higher-order mode, the comb is switched on via modulation instability [8]. The comb is coherent and stable over long term in a temperature-controlled environment. Figure 1a) shows a schematic for the Kerr comb generation while Fig. 1b) shows a typical spectrum, before spectral flattening, that covers the C-band with the lines spaced every 3 FSRs (690GHz). The comb is in a low-noise state, as indicated by the electrical spectrum analyzer [6], see Fig. 1c). Next, to emulate a transmission scenario, we equalized the Kerr comb lines in power using a pulse shaper and an EDFA [2]. The comb lines were modulated with 20Gbaud polarization multiplexed 16QAM data using an arbitrary waveform generator (AWG) and a polarization multiplexing stage, see Fig. 1d). We assessed the bit-error rate (BER) vs. optical signal-to-noise ratio (OSNR) at 0.1nm bandwidth in back-to-back for every line using an additional noise-loading stage. The receiver consisted of a tunable local oscillator laser, a coherent receiver and a 23GHz real-time oscilloscope. The BER was calculated after processing the received data by offline DSP containing a decision-directed least mean square equalizer and phase tracker.

Figure 1e) shows that a different OSNR is required depending on the line in order to achieve the same BER of $10^{-3}$. Although the penalty difference is less than 1dB, it seems to follow roughly the same wavelength variation as the equivalent linewidth between the measured comb line and the local oscillator. The equivalent linewidth was calculated digitally from the DSP data in the 2 – 20MHz region of the FM noise using the method described in the following section. In order to assess whether the linewidth variation is a feature inherent to the comb we realized a careful measurement of the FM noise spectrum of each line using self-delayed heterodyne coherent detection [9].
3. Line-resolved linewidth measurements

The phase of an optical light wave can not be measured directly. By using an interferometric setup, such as the one shown in Fig. 2a), one can however measure its drift over time. In the measurement setup, after filtering out one comb line and attenuating it to a fixed level, we split the light in two arms that we then fed into a coherent receiver. One of the arms was delayed by a time longer than the coherence time of the pump laser using a fiber. It was also shifted by 27 MHz using an acousto-optic modulator to avoid artifacts from the DC block in the coherent receiver. The outputs of the coherent receiver were connected to two ports of a 6.25 GS/s real-time oscilloscope from which we could extract the signal phase after compensating for the IQ imbalance in the receiver and the linear phase shift caused by the acousto-optic modulator. To extract useful information about the phase noise, we then calculate the FM noise spectrum, i.e. the power spectral density of the instantaneous frequency, $S_{\text{FM}}$ [7]:

$$S_{\text{FM}}(\nu) = \langle |\tilde{F}[f(t)]|^2 \rangle = \left| \langle \frac{1}{2\pi} \frac{\partial \phi(t)}{\partial t} \rangle \right|^2.$$

(1)

In Eq. 1, $\tilde{F}$ denotes the Fourier transform while $\langle \rangle$ denotes ensemble averaging. In the case of a laser line with Lorentzian shape, $S_{\text{FM}}$ is flat and one can directly extract the combined linewidth by multiplying it with $\pi$ [7]. In the context of optical communications, the low frequency components of the phase noise can typically be tracked and compensated for successfully in DSP. The noise floor, or the Lorentzian-like linewidth therefore has a more significant effect on the final BER [9].

Both the reference pump laser as well as the comb lines were then measured using the setup in Fig. 2a). The FM noise spectrum of the tunable external cavity laser used to pump the microresonator can be seen in Fig. 2b) together with one of the comb lines. The pump laser noise floor is at 3000 Hz$^2$/Hz, translating to a Lorentzian-like combined linewidth of approximately 16 kHz in close agreement to the specifications. Since the phase noise of the comb lines did not differ significantly from each other, an alternative comparison method was needed. For this purpose two spectral regions were selected and averaged over: 0.1 – 1 MHz and 2 – 20 MHz, shown as shaded areas in Fig. 2b).

To ensure that the setup was wavelength-independent, the reference pump laser was also tuned and measured at all wavelengths matching the lines of the frequency comb. Additionally, to assess the effect of the pump laser, the microresonator was also pumped with a different, higher linewidth, external cavity laser. The results can be seen in Fig. 2c) where the FM noise spectrum values at the chosen frequencies have been multiplied by $\pi$ to get a Lorentzian equivalent linewidth. The tuned reference laser is shown as black circles. The comb lines generated using the same laser are marked with blue circles while the comb lines generated by the high linewidth laser are marked in red.

![Diagram](image.png)

Fig. 1. a) Setup for generating and flattening the frequency comb. b) Optical and c) electrical power spectrum of the comb running in a mode with low amplitude noise. d) Noise loading setup for measuring implementation penalty. e) The equivalent combined linewidth of the comb and a free-running LO shown together with the corresponding OSNR requirement for a 16QAM system operated at a receiver BER=$10^{-3}$. The theoretical OSNR limit at 18.6 dB means that the penalty is around 2 dB. The inset shows a constellation diagram for the comb line at 1538 nm at BER=$10^{-3}$.
4. Conclusions

We have characterized the FM noise of a normal dispersion Kerr comb providing information about the comb linewidth in a frequency-resolved manner. In the low-frequency region (corresponding to self-heterodyne ESA-based beatnote linewidth measurements), our results indicate that the linewidth of the comb lines is largely dictated by the pump laser and does not vary significantly from line to line. However, the instantaneous linewidth in the short temporal scale, that is relevant for phase tracking in coherent communication systems, displays an asymmetric behavior with respect to wavelength. The linewidth can vary by more than a factor of ten, depending on the pump laser. While different types of Kerr combs might have different scaling effects, further investigations need to be done for a complete physical understanding. These results extend previous observations of equal linewidth across the bandwidth of Kerr combs and call for extra caution and laser pump requirements when using higher-order constellations.

5. References


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Active feedback stabilization of normal-dispersion microresonator combs

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Active Feedback Stabilization of Normal-Dispersion Microresonator Combs

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Silicon nitride microresonators provide a potentially CMOS-compatible platform for optical frequency comb generation [1]. As in any high-Q microcavity, thermal effects strongly affect the dynamics. This is particularly true considering that typical continuous wave pump powers for microresonator comb generation operate in the 20-35 dBm range. Temporal bright solitons for instance are generated when the pump is on the red side of a resonance, a situation that is challenging to attain owing to the thermo-optic coefficient of most microresonator materials [2]. Active stabilization has been successfully demonstrated by fixing the soliton power at a certain stage in the initialization process [3] or by careful power kicking and detuning of the pump laser [4].

Normal-dispersion combs aided by transverse mode interactions [5] can instead be pumped on the thermally stable blue side of the microresonator resonance and have recently been shown to provide unusually high conversion efficiency into the comb [6]. Nevertheless, thermal drifts on a long time scale remain. Here, we demonstrate active feedback stabilization of a normal-dispersion silicon nitride microresonator comb by using the comb power (excluding the pump) as the control parameter to tune the frequency of the pump laser (Toptica CTL) via a feedback loop. Hence, the comb output power can be kept stable over long term (<0.05 dB comb power variation over an eight-hour time period, Figs. a) and b). Additionally, since the initialization process of normal-dispersion combs requires no pump-power changes, a chosen comb state can be accessed directly from an off-state by picking a comb power set point as the target for the regulator.

Since the pump power is kept constant, different set points represent different conversion efficiencies. We observe that as the set comb power increases, so does the comb bandwidth (Fig. c), while maintaining a low noise state (Figs. d and e). This situation is in sharp contrast to bright temporal solitons [7] and verifies that the bandwidth in normal-dispersion combs scales with the conversion efficiency [8]. In the next set of experiments, we plan to verify the pulse compression capabilities and measure the effective cavity detuning.

These results are important from an applications perspective, allowing a simpler start-up condition while at the same time permitting to run experiments over long term. They also demonstrate that the comb power-bandwidth relation is fundamentally different in combs operating in the normal-dispersion regime, - a crucial aspect for optical communication applications [9, 10].

References
High-order coherent communications using mode-locked dark pulse Kerr combs from microresonators


High-order coherent communications using mode-locked dark-pulse Kerr combs from microresonators

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Microresonator frequency combs harness the nonlinear Kerr effect in an integrated optical cavity to generate a multitude of phase-locked frequency lines. The line spacing can reach values in the order of 100 GHz, making it an attractive multi-wavelength light source for applications in fiber-optic communications. Depending on the dispersion of the microresonator, different physical dynamics have been observed. A recently discovered comb state corresponds to the formation of mode-locked dark pulses in a normal-dispersion microcavity. Such dark-pulse combs are particularly compelling for advanced coherent communications since they display unusually high power conversion efficiency.

Here, we report the first coherent transmission experiments using 64-quadrature amplitude modulation encoded onto the frequency lines of a dark-pulse comb. The high conversion efficiency of the comb enables transmitted optical signal-to-noise ratios above 33 dB while maintaining a laser pump power level compatible with state-of-the-art hybrid silicon lasers.

Replacing a large number of lasers in wavelength-division-multiplexed (WDM) optical communications systems with an optical frequency comb has always been an attractive idea. Until recently, demonstrations focused on broadened mode-locked lasers⁴-⁵ and electro-optic frequency combs³ made using a cascade of phase and intensity modulators⁴. Electro-optic comb generators can use a single high-quality laser as a seed and replicate its properties to several channels. Increasing the bandwidth further is possible by using nonlinear broadening⁶-⁷, allowing for lighting more WDM channels. Optical frequency combs have an intrinsically stable frequency spacing that enables transmission performance enhancements beyond what is possible with free-running lasers. Recent demonstrations include multi-channel nonlinear pre-compensation⁸ as well as the possibility to decrease the inter-channel guard-bands for an increased total spectral efficiency⁹,10. Another exciting prospect for using a frequency comb as multi-carrier light source in WDM systems is the possibility to relax the resource requirements at the receiver by implementing joint impairment compensation and tracking for multiple data channels¹¹,12. This aspect exploits the broadband phase coherence of the frequency comb and is therefore impossible to carry out using a multi-wavelength laser array.

In order to implement practical WDM transmitters while minimizing the number of discrete components, photonic integration will be needed. It is however challenging to attain broad bandwidth and high-
powered chip-scale frequency combs with similar levels of performance as in the demonstrations above. Initial attempts have included silicon-organic hybrid modulators, quantum-dash mode-locked lasers and gain-switched laser diodes. A CMOS-compatible platform that shows great promise in this direction is the microresonator frequency comb implemented in silicon nitride technology. Microresonator frequency combs (or Kerr combs) use the Kerr effect in an integrated microcavity to convert light from a continuous-wave pump laser to evenly spaced lines across a wide bandwidth. The first data transmission demonstrations were performed using on-off keying. It was however soon recognized that the performance of microresonator combs is sufficiently high to cope with the requirements in terms of frequency stability, signal-to-noise ratio (SNR) and linewidth of modern coherent communication systems. Recent demonstrations have therefore included advanced modulation formats and long-haul communication systems. The discovery of dissipative Kerr solitons in microresonators and associated stabilization schemes has opened a path forward to control the bandwidth and number of comb lines with great precision. One of the most recent experiments has achieved impressive aggregate data rates using two SiN microresonator combs spanning the lightwave communication C & L bands. Using thermal control, tuning of the central frequency of the combs has allowed the use of a matched comb at the receiver as a multi-wavelength local oscillator.

Microresonator frequency combs are however complex systems that permit several different regimes of low-noise operation. The coherent communications demonstrations thus far have mainly focused on combs operating in the bright soliton and coherent modulation instability regimes. Recent experiments have revealed a mode-locked state when the cavity exhibits normal dispersion. This comb state corresponds to circulating dark pulses in the cavity and it might be of significant interest for coherent data transmission in WDM systems. These dark-pulse combs have experimentally measured power conversion efficiencies between the pump and the generated comb lines above the 30% mark for combs spanning the C band, i.e. significantly higher than what can be fundamentally obtained with bright soliton combs. If these powers could be harnessed, such dark-pulse combs could either decrease the pump power requirements or enable WDM transmitters with higher SNRs. This is a relevant matter as modern communication systems move toward ever more advanced, higher-order modulation formats. These formats contain increasing amounts of encoded data per transmitted symbol, which results in increased requirements in the received SNR.

In this work, we show what we believe is the first coherent WDM transmission experiment done with a dark-pulse Kerr comb. We use a SiN-based microresonator that produces comb lines satisfying the optical signal-to-noise ratio (OSNR) requirements for modern coherent communication formats. The high OSNR per channel is enabled by the high internal conversion efficiency of the comb, which reaches above 20% - in line with previous observations. Using off-chip pump powers below 400 mW, we demonstrate 80 km data transmission with 20 channels. Each channel contains data modulated using 20 Gbd 64-quadrature amplitude modulation (QAM) resulting in an aggregate data rate of 4.4 Tb/s. This demonstration corresponds to the highest order modulation format shown with any integrated comb technology to date.
Results

Microresonator-based frequency comb generation. A silicon nitride-based microresonator is used for the comb generation. The resonator’s 100 µm radius results in a free spectral range of about 230 GHz. The ring waveguide features a designed width and thickness of 2 µm and 600 nm, yielding normal dispersion in the C band. Fabrication details are described in ref. 39. The intrinsic Q-factor is measured to be around 1.6 million. For the comb generation, the microresonator is pumped by a tunable external cavity laser with less than 10 kHz specified linewidth. Before reaching the microresonator, the laser light is amplified and filtered ensuring an off-chip continuous-wave pump of 25.6 dBm. Figure 1a shows a sketch of the setup used for the comb generation. The fiber-to-chip coupling losses at high pumping powers are estimated to be 5 dB per facet. The microresonator is equipped with both a through and a drop port, with the latter being used for assessing the intracavity waveform. As the coupling between the resonator and the through port is stronger, the comb obtained at the through port is used for the communication experiments (see Methods).

![Figure 1](image)

Figure 1 | Frequency comb operation. (a) Setup for generating and measuring the comb state. The amplified spontaneous emission noise generated by the erbium-doped fiber amplifier (EDFA) is removed using a 200 GHz band-pass filter centered at the pump wavelength. The comb state is stabilized against long-term drifts by selecting a line 460 GHz above the pump frequency using a second 200 GHz band-pass filter. (b) Comb spectrum measured at the through port with 0.1 nm resolution. The 20 comb lines in the shaded region between 1531nm and 1566 nm are used for the communications experiment (The C band is typically specified between 1530 nm and 1565 nm). (c) Directly measured and (d) reconstructed intracavity time-domain waveforms using the drop port of the microresonator. (e) The comb line spacing measured with 1 kHz RBW.

The comb is initialized by tuning the pump laser wavelength into a resonance located at 1540 nm from the thermally stable blue side. To monitor the running comb state, a photodiode is placed after an optical band-pass filter centered at a newly generated comb line around 1536 nm, see Fig. 1a. By using the photodiode output as feedback to the laser wavelength setting, it is possible to start the comb by placing the laser close to resonance and simply initializing the lock. This way, the pump will stop sweeping at the moment the comb is in the desired state. While locking is not necessary to keep the comb running over
several hours, the feedback loop ensures that laboratory environmental changes do not cause the spectrum to change significantly over this time. The spectrum of the generated comb at the through port, displayed in Fig. 1b, shows the characteristic envelope of dark pulses. To verify that the intracavity comb state corresponds to a circulating dark pulse, two separate time-domain measurements are taken at the device’s weakly coupled drop port. A direct measurement performed using a 500 GHz bandwidth optical sampling oscilloscope results in the waveform shown in Fig. 1c indicating square-like pulses. An effectively higher bandwidth, but indirect, measurement is also taken by measuring the comb lines’ spectral phase after which the time-domain picture is reconstructed as shown in Fig. 1d. The Methods section contains a more detailed description of the measurement and reconstruction procedure. Additionally, the comb line spacing stability was measured using electro-optic downconversion. The beat note (see Fig. 1e) displays a clear peak (FWHM < 30kHz) with > 50dB SNR indicating stable mode-locking operation.

**Figure 2** | Transmitter description. (a) Schematic of the transmitter. WSS: wavelength-selective switch; AWG: arbitrary waveform generator. (b) Optical spectrum after pump line attenuation and the single-stage amplifier. The 20 lines of interest all have an OSNR of 35 dB or more. (c) Optical spectrum of the WDM signal before being sent to the link with all channels having above 33 dB OSNR.

**Optical data modulation.** The microresonator comb described in the previous section is now shown to support data transmission with advanced modulation formats. To ensure maximum comb line powers for the data transmission experiment, we used the dark-pulse comb at the through-port as light source for the transmitter. At this port, the total fiber-coupled comb power is roughly 28 mW (out of which about 8.6 mW is in the newly generated comb lines) leading to an on-chip comb power conversion efficiency above 20% and a flattened net conversion efficiency of 1.5%, see the Methods section for calculation details. Following the chip itself, a 200 GHz notch filter centered at the pump wavelength attenuates the central comb line, allowing for an efficient operation of the following optical amplifier. For the 80 km single-span WDM experiment, the transmitter schematic is shown in Fig. 2a with the initially filtered and amplified comb spectrum visible in Fig. 2b. Following amplification, the power in the comb lines is split into two arms (marked as odd and even in the setup) using a commercially available wavelength-selective
switch (WSS). This keeps the number of lines going into each modulator at ten. The WSS is also used to equalize the powers among the comb lines in each arm separately. Each modulator is driven by signals generated in an arbitrary waveform generator (AWG). The AWG is programmed to generate two independent random 64QAM signals using square pulses, each carrying 6 bits per symbol at a rate of 20 GBd. The random symbol sequence has a length of $2^{16}$ symbols and an oversampling of three since the AWG is operated at 60 GS/s. To mitigate non-idealities in the digital-to-analog converters (DACs) as well as the modulators, digital pre-compensation is applied on the signal in the AWG. Following the modulators, a split-and-delay polarization multiplexing stage, using a $\geq 1$ m long arm corresponding to $\geq 100$ data symbols, is used to emulate a polarization-multiplexed transmitter. By using both polarizations, the system capacity is effectively doubled. Both arms are then recombined and flattened using a second WSS before being sent to the link. The second flattening step will translate the power differences in the two arms (caused by the comb envelope and initial flattening step) into the slightly varying noise floor seen in Fig. 2c.

Transmission results. After the 80 km long standard single-mode fiber link (with about 16 dB propagation loss) there is a polarization-diverse single channel coherent receiver, see Fig. 3a. A tunable external-cavity laser (with linewidth below 100 kHz) is used as a local oscillator, allowing for the reception of one data channel at a time using a 23 GHz bandwidth real-time oscilloscope operating at a sampling rate of 50 GS/s. Standard digital signal processing (DSP) algorithms are subsequently run offline and are described in more detail in the Methods section. A representative example of a received constellation is presented in Fig. 3b. To ensure optimal signal power in the fiber, data is recorded for multiple launch powers. The results in Fig. 3c correspond to the optimum case, with a launched signal power of 3 dBm per channel. Higher power levels incur nonlinear distortion, whereas lower powers lead to a system performance limited by noise. The resulting BERs are calculated by comparing the decoded bit stream with the transmitted one. All comb lines provide sufficient BER margin for transmission over 80 km fiber. The resulting BER values allow the application of a hard-decision staircase forward error correcting (FEC) code with an overhead of 9.1% to reach a final post-FEC BER below $10^{-15}$ yielding an aggregate data rate of 4.4 Tb/s.

The final experiment corresponds to a configuration where there is no transmission fiber (“back-to-back”). This experiment serves two purposes. First, it allows quantifying the performance of the transmitter/receiver with respect to an idealized situation where only additive white Gaussian noise is considered (theory). Second, it allows for distinguishing penalties coming from the comb source and from the transceiver subsystems by comparing the results with a similar measurement performed using a stand-alone laser. The measurement is performed on a channel-by-channel basis by measuring the received BER for varying OSNR. The setup and measurement details are described in the Methods section. As shown in Fig. 3d, at the chosen BER threshold of $7\cdot10^{-3}$ the comb lines require a slightly higher received OSNR (between 0 dB and 0.5 dB) compared with the free-running laser system. Both the comb lines and reference lasers show that an increase in OSNR by 3 dB with respect to the theoretical prediction is required to reach the same target BER value (this is often referred to as implementation penalty). Such a deviation from the theoretical case is expected for advanced modulation formats with high symbol rates, where the limited effective number of bits in both the transmitter and the receiver electronics impair the transmitted signal. These results indicate that the microresonator comb source does not significantly
impair the transmission link performance and is therefore a suitable light source for high-order coherent optical communication systems.

**Figure 3 | Receiver and results.** (a) Schematic of the receiver including the blocks in the offline DSP. (b) Received constellations from the 1531 nm line. The constellations are displayed for both polarizations at corresponding bit error ratios (BERs) of $3.29 \times 10^{-3}$ and $2.53 \times 10^{-3}$. (c) Received BERs averaging both polarizations for five batches for each comb line after the 80 km long transmission link. The error bars show the BER values for the best and the worst batch for each wavelength. The wavelength-dependent BER variations are within expected levels considering the transceiver components. (d) Noise-loading measurements for three selected comb lines and comparison measurements using a free-running laser. The black full-drawn curve corresponds to the theoretical additive white Gaussian noise channel case assuming Gray coding. The dashed curve shows the maximum permitted BER allowing for a 9.1% overhead for the implementation of a hard-decision staircase FEC code. All measurements were performed with random data modulated using 20 Gb/d PM-64QAM.

**Discussion**

In summary, we have presented the first demonstration of coherent WDM communications using dark-pulse microresonator combs. We have shown the highest-order modulation format demonstrated using any integrated comb source. An important aspect of this study is that it illustrates that the favorable power conversion efficiency of dark-pulse combs can be used in practice to reach channel OSNRs > 33 dB while maintaining an on-chip pump power in the order of a few hundred mW.

While in this exploratory study the setup complexity was extensive and the spectral efficiency relatively low (about 0.95 b/s/Hz), these are not fundamental limits of the capabilities of microresonator comb-based transmission systems. In this section, we analyze the fundamentally achievable OSNRs per wavelength channel for optimized dark-pulse combs. We demonstrate that the achievable OSNR levels are compatible with state of the art hybrid silicon tunable lasers, which feature optical linewidths of 15 kHz and power levels in the order of 100 mW. The resulting OSNRs are sufficiently high to encode higher-order modulation formats while leaving sufficient margin to allow for losses when the comb is co-integrated with other active components in a transmitter system.

For data transmission purposes, the weakest line power in the comb will fundamentally dictate the minimum achievable OSNR per line at the transmitter. The line’s power level can be maximized by jointly
optimizing the group velocity dispersion coefficient and coupling rate of the microresonator. We ran an optimization process (see Methods) of a dark-pulse comb spanning the C band by sweeping these two parameters while keeping the microresonator losses, pump power, line spacing and nonlinear parameter fixed. The pump power was fixed to 100 mW and a line spacing of 100 GHz was selected that is more compatible with WDM standards. Assuming that challenges involving propagation loss and multi-mode behavior can be handled, a larger resonator with lower FSR could in principle result in a dark-pulse comb covering the C band with a line spacing closer to 100 GHz. Recent demonstrations indicate that this is indeed possible. The result of the optimization process is shown in Fig. 4a and it shows that an optimum state can be found for a moderate amount of normal dispersion and a strongly coupled microresonator. The fabrication feasibility of these parameters is discussed and verified in Supplementary note 1. The weakest line power within the C band appears at the -10 dBm level and the resulting comb is shown in Fig. 4b. The comb lines have a power level varying between 51 dB and 71 dB above the quantum noise limit (corresponding to around -61 dBm for a single polarization and 0.1 nm bandwidth). Supplementary note 2 contains results using the same optimization process for combs with 50 GHz line spacing as well as combs covering both C and L bands.

![Image](image_url)

**Figure 4 | Dark-pulse comb optimization for modern communications formats.** (a) Simulation results showing the minimum comb line powers as the waveguide’s group velocity dispersion coefficient, $\beta_2$, and the ring waveguide coupling constant, $\theta$, are varied. The comb spacing is fixed at 100 GHz. The peak value marked with X at -10 dBm is achieved for $\beta_2=350$ ps$^2$/km, $\theta=0.011$ and a detuning of $\delta_0=0.0445$, yielding a comb with above 50% power conversion efficiency and a flattened net conversion efficiency of 4.4%. (b) Optical spectrum of the optimized dark-pulse comb with line powers in the C band between 51 dB and 71 dB above the quantum noise limit at 0.1 nm resolution. (c) Theoretical OSNR requirements at the receiver side for a variety of modern communication formats and symbol rates assuming additive white Gaussian noise and Gray-level coding. For a single-polarized signal, the limits will be the same assuming that the noise is only measured in that polarization. We note that a doubling in symbol rate or modulation format results in roughly a 3 dB increase in the OSNR requirement.

Receiver OSNR requirements for advanced modulation formats and symbol rates are displayed in Fig. 4c. For example, 50 GBd PM-64QAM would require an OSNR at the receiver side of 26 dB. Assuming the WDM
transmitter is amplified with an EDFA with a noise figure of 4 dB and considering 5 dB implementation penalty due to the limited effective number of bits at the transmitter and receiver yields a margin of 16 dB for the weakest comb line. If the same microresonator were used as a source for signals in both polarizations, a further 3 dB should be deducted. The remaining 13 dB margin are to be split between optical losses at the integrated transmitter (containing multiplexers and modulators) and the effects of the following link (owing to added noise by further amplification as well as nonlinear effects\textsuperscript{51}). Given the high available power in the central lines, these can carry data using even higher order modulation formats, potentially targeting PM-256QAM. A transmitter scheme where the power in the stronger lines can be exploited in this manner is discussed in Supplementary note 3. Considering recent advances in silicon photonic circuits\textsuperscript{49}, we envision that the level of performance obtained in the experiments reported here might be obtained for a fully integrated transmitter system featuring an optimized dark-pulse covering the whole C band using a 100 mW pump source.

**Methods**

**Indirect time-domain measurements.** The indirect time-domain measurement was done by sending the frequency comb through a WSS and an EDFA to a non-collinear optical intensity autocorrelator\textsuperscript{33,42}. By selecting three lines at a time in the WSS and adjusting their phases incrementally to achieve minimum pulse width in the autocorrelation measurement, we could extract the relative phases across all comb lines within the bandwidth of the EDFA and the pulse shaper. The measurements were performed using both the drop and the through ports of the microresonator with the resulting phases overlapping for all measured lines except for the pump line. Using a known mode-locked laser delivering transform-limited pulses as a reference, the dispersive effects of the fibers in the EDFA, the WSS and the fiber connections were measured and compensated for. While most of the power in the comb was within the bandwidth of the EDFA and the pulse shaper, the spectral phases of the lines outside were estimated using linear extrapolation.

**Line spacing stability measurements.** As the frequency difference between the comb lines was too large for our photodiodes' bandwidth, a direct measurement was not possible. Instead, the spectrum between the lines was filled using electro-optic (EO) modulation\textsuperscript{43}. We selected two central comb lines with a 2 nm optical bandpass filter (at 1539.8 nm and 1541.6 nm) and generated EO-modulated lines between them using an RF-oscillator operating at 25.1 GHz. The beat note between two resulting EO-comb lines at 1540.8 nm (one originating from each original dark-pulse comb line) was then filtered out using a 0.25 nm filter and recorded using a real-time oscilloscope. From 1 ms of recorded data, we could thereby retrieve the spectral stability of the comb line spacing with 1 kHz resolution by standard Fourier processing.

**Microresonator operation.** The chip containing the microresonator was kept on a piezo-controlled positioning stage stabilized using a standard laser temperature controller at 18°C with less than 0.01°C variation. Light was then coupled into the bus waveguide using a lensed fiber. On the chip, the coupling between the bus waveguide and the ring was set by their 300 nm wide gap. As the drop port gap (at 1000 nm) was much larger, the power coupling to that port is estimated to be more than 10 times weaker (see Supplementary note 1). The roundtrip losses owing to light getting coupled out of the resonator is therefore expected to be dominated by the through port.

As the comb generation process depends strongly on the presence of modal coupling\textsuperscript{52}, the spectral envelope is sensitive to slight fabrication variations. Out of three similar devices, two produce dark pulse combs using a 1540 nm pump. The tolerances (and thereby the device yield) is however expected to be improved using techniques enabling post-fabrication tuning, for example using controllable mode interactions as described in ref. 53.

**Comb power conversion efficiency calculation.** To estimate the power conversion efficiency of the comb generation process, both the comb state and the reference off-resonant state have to be compared under identical pump power and polarization conditions. The conversion efficiency is calculated by comparing the sum of the power in the generated comb lines in the on state with the power of the pump line in the off state\textsuperscript{26}. 

To estimate the useful net conversion efficiency, one can perform a similar calculation. Instead of summing the power of all the newly generated comb lines, one should instead take the power in the weakest line within the bandwidth of interest (the C band in this case) and multiply that with the number of lines present within the same bandwidth (20). Comparing this power with pump line power in the off state will yield the effective flattened on-chip conversion efficiency.

**Digital signal processing algorithms.** To decode data from the recorded complex waveforms, receiver non-idealities, link effects and transmitter non-idealities had to be handled, in that order. The following steps describe the DSP operations:

1. Receiver impairments were handled by first compensating for relative delays owing to differences in the RF cable lengths in the I and the Q arm of the coherent receiver. This was followed by an IQ imbalance compensation using Gram-Schmidt orthogonalization\(^5\). After this, the waveform was resampled to twice the symbol rate. All these steps were performed independently for each polarization.

2. In the case of the 80 km long transmission, chromatic dispersion of the single-mode fiber link was removed using a static filter implemented in the frequency domain individually for each polarization.

3. A decision-directed least-mean-square equalizer was used for signal equalization and polarization demultiplexing. This step also included an FFT-based frequency offset estimator\(^55\) and a blind-phase-search component to compensate for relative phase drifts between the signal carrier and the local oscillator. The equalizer contained 25 taps and was trained by letting it run over the waveform four times with decreasing step length.

4. A second Gram-Schmidt orthogonalization was performed individually on each polarization to compensate for small modulator bias errors.

5. Finally, the BER was calculated by comparing the bit sequence decoded from the received symbols with the originally transmitted one. The total received sequence from which the BER was calculated contained more than 9 million bits.

**Noise-loading measurements.** The noise-loading measurements allow comparing the performance of single channels with theory to extract quantitative penalties accrued owing to the system implementation (occurring for example due to the limited resolution in the transmitter digital-to-analog converters and receiver analog-to-digital converters). To isolate these penalties from those coming from the comb source, separate measurements were taken with individual comb lines as well as a reference laser. For the evaluation of our transmitter and receiver system, a single tunable external-cavity laser was therefore used. The reference laser corresponded to a standard (below 100 kHz linewidth) communications laser with above 15 dBm output power, nominally identical to the local oscillator. The reference laser was connected directly to the modulator in one of the arms in Fig. 2a. Following the polarization multiplexing stage, the channel was loaded with noise by successive attenuation and amplification. Finally, the channel was then received resulting in the BER vs. OSNR plots in Fig. 3d yielding a $\leq 2.5$ dB system penalty with respect to theory at BER=$7\times10^{-3}$. To make an equivalent measurement with the comb source meant selecting single comb lines and amplifying them to the same 15 dBm power level before performing data modulation.

**Dark-pulse comb simulations.** The comb state simulations were performed using the Ikeda map\(^57-59\). This method allows for including the pump noise in every round trip and to quantify the resulting OSNR per spectral line. The method involves simulating the coupling between the bus waveguide and the ring separately from the light propagation inside the ring cavity. The coupling region was simulated as a lossless directional coupler\(^60\) whereas the propagation in the ring was implemented with the nonlinear Schrödinger equation. In the coupling step, a continuous wave pump laser (with 100 mW fixed power) was coupled in together with quantum noise equivalent to one photon per spectral bin\(^58\). The power coupling coefficient $\theta$ was swept to produce the map in Fig. 4a. Additionally, to account for the detuning, $\delta_0$, between the pump laser wavelength and the resonance center, a corresponding phase shift was applied to the current intracavity field. The propagation simulation was performed using a split-step nonlinear Schrödinger equation solver, where the linear step (with fixed power loss parameter $\alpha=0.1$ dB/cm, corresponding to an intrinsic quality factor of 1.8 million, and swept group velocity dispersion coefficient, $\beta_2$) and the nonlinear step (with fixed nonlinear parameter $\gamma=2$ W$^{-1}$m$^{-1}$) were performed iteratively in 16 steps along the ring.

To ensure that the comb state converged to a dark-pulse comb, the intracavity field was initialized with a square wave whose upper and lower power values correspond to the continuous-wave bi-stability solutions (ref. 33). Once the field inside the microresonator had converged to a steady-state, the result was analyzed. For each combination of $\beta_2$ and $\theta$, the detuning parameter and initial square pulse width giving the best comb state were chosen. While there are several metrics by which combs can be evaluated, in this work we optimized the parameters for maximum power in the weakest line within the C band\(^41\).
Data availability. Code to plot the figures that appear in this paper as well as raw data and corresponding processing scripts for the main 80 km transmission results are available online here.

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High-order coherent communications using mode-locked dark-pulse Kerr combs from microresonators: Supplementary material

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Supplementary note 1: Fabrication feasibility of the optimized comb

In order to evaluate the fabrication feasibility of the waveguide suggested in Fig. 4 in the main paper, we performed cross-section simulations using a finite element method-based mode solver (COMSOL). We calculated the chromatic dispersion for a variety of realistic rectangular waveguide geometries for wavelengths around \( \lambda=1550 \) nm. Figure S1 shows that using rectangular geometries, it is indeed possible to achieve the optimal dispersion value for the fundamental waveguide mode using several width and height combinations. The material dispersion for silicon nitride used in the simulations was taken from refs. 1,2. As the comb initialization strongly depends on the interaction between two co-polarized modes3, the part of the valid contour supporting exactly two X-polarized modes has been highlighted.

In the following, we illustrate that the power coupling parameter, \( \theta \), obtained in the simulations is also achievable in realistic geometries. Using the same mode solver as above together with standard coupled mode equations it is possible to evaluate the coupling constant \( \kappa \) between the fundamental modes of two adjacent waveguides4,5. Figure S2a illustrates how this constant is varying for a twin waveguide system.

![Figure S1 | Dispersion of rectangular waveguides](image-url)
(with 1000 nm x 600 nm waveguides) as the gap distance is swept. This geometry provides a solution to optimize the coupling ideality when the microresonator sustains more than one mode. Depending on the achievable minimum gap during fabrication, this will translate into a varying required length of the coupling region as illustrated in Fig. S2b. As the resonator circumference for a 100 GHz resonator is more than one millimeter, all the shown coupling lengths are feasible with standard microlithographic tools.

**Supplementary note 2: Comb bandwidth and line spacing**

In the discussion section of the main paper, we found the optimum 100 mW-pumped C-band-spanning 100 GHz spaced dark pulse comb. It is however possible to optimize for other scenarios as well. Dark pulse combs can be designed to operate with different pump powers, narrower spacing as well as covering a wider bandwidth. In Fig. S3a we show corresponding optimal comb states for 50 GHz and 100 GHz combs covering the C-band (1530 nm – 1565 nm) and C+L bands (1530 nm – 1610 nm). The optimization process was identical to the one done in the main paper as described in the main Methods section. As predicted for other microresonator comb states, increasing the number of generated comb lines will lower the reachable conversion efficiency as well as the power available in the weakest line.

In Fig. S3b we have varied the pump power showing that dark pulse comb states remain viable multi-wavelength light sources with pump powers below 20 dBm. A 3 dB decrease in pump power translates to a 4-6 dB decrease in the weakest line power. At the low power end, a 25 mW pump will be enough to give a C-band spanning 100 GHz comb a minimum line power of -20 dBm. This corresponds to 38 dB above the quantum-limited noise floor. In principle, this is enough margin to allow for PM-64QAM modulation, assuming the surrounding equipment does not further decrease the limit. Should further margins be required, one can relax the received OSNR requirement by using more advanced, soft-decision, error correction schemes.
Supplementary note 3: Conversion efficiency utilization

In the main paper, to overcome the limitations of using two modulators used for the even and odd channels, the power in all comb lines was equalized and amplified before data modulation. This design does not make efficient use of the extra power in the strongest comb lines however, thereby decreasing the net conversion efficiency from >20% to 1.5%. In a scenario where one has access to a separate modulator for each comb line, alternative designs exist that can make better use of the available power. Figure S4a displays an example system where the available power is instead translated to higher optical signal-to-noise ratio in the central channels. In those channels, the high OSNR would allow for data to be encoded with higher order modulation formats or lower coding overhead. Note that the channel powers...
will still be equal as they are flattened after the first transmitter EDFA. The design does not include active components (such as amplifiers) between the comb source and the modulators, thus simplifying future integration. The sketched system connects the optimum simulated comb from the main paper (Fig. 4b) with transmitter components. By assuming combined multiplexing, demultiplexing and modulator losses of 16 dB and an EDFA noise figure of 5 dB, the dual-polarized data-carrying channels will have OSNRs between 27 dB and 50 dB, see Fig. S4b. In this scenario, the weakest lines have slightly lower OSNR than the ones demonstrated in the main paper. The example system however simultaneously allows efficient usage of the comb’s conversion efficiency while requiring no active components until the final fiber connection.

Figure S4 | Comb-based transmitter sketches. (a) Setup with flattening stage after modulation allowing the line power differences to be translated to OSNR differences in the transmitted channels. (b) Resulting OSNR (at 0.1 nm resolution) variation for the simulated comb in the main paper assuming 16 dB losses in the transmitter stage and a 5 dB noise figure EDFA.

Supplementary references

Paper E

Triply resonant coherent four-wave mixing in silicon nitride microresonators

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Triply resonant coherent four-wave mixing in silicon nitride microresonators

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Generation of multiple tones using four-wave mixing (FWM) has been exploited for many applications, ranging from wavelength conversion to frequency comb generation. FWM is a coherent process, meaning that its dynamics strongly depend on the relative phase among the waves involved. The coherent nature of FWM has been exploited for phase-sensitive processing in different waveguide structures, but it has never been studied in integrated microresonators. Waveguides arranged in a resonant way allow for an effective increase in the wavelength conversion efficiency (at the expense of a reduction in the operational bandwidth). In this Letter, we show that phase shaping of a three-wave pump provides an extra degree of freedom for controlling the FWM dynamics in microresonators. We present experimental results in single-mode, normal-dispersion high-Q silicon nitride resonators, and numerical calculations of systems operating in the anomalous dispersion regime. Our results indicate that the wavelength conversion efficiency and modulation instability gain in microcavities pumped by multiple waves can be significantly modified with the aid of simple lossless coherent control techniques. © 2015 Optical Society of America

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The effect of four-wave mixing (FWM) has enabled a variety of ultrafast photonics applications, including high-speed sampling, switching, wavelength conversion, and amplification [1,2]. The FWM effect can become very efficient when using resonating systems [3] when the input waves are matched to the resonator cavity’s longitudinal modes. This comes at the expense of a reduction in the operational bandwidth owing to an inherent tradeoff between resonance linewidth and conversion efficiency (although this can be partly alleviated by using an arrangement of coupled resonators [4]). Nevertheless, resonant FWM has been used in practice for wavelength conversion of high-speed data signals [4] and threshold-less comb generation with two [5,6] and several pumps [7]. Synchronous pumping of nonlinear fiber loops has been used for optical data buffering [8]. Nonlinear effects, including the breaking of time-reversal symmetry [9] and the generation of dispersive waves [10], have been reported in fiber cavities.

If three or more waves are input to the nonlinear medium, the gain dynamics of the mixing process, whether resonant or not, depends on their relative phases [11]. While this phase-sensitive process has been widely studied in different fiber configurations [11, 12], the phase-sensitive dynamics of FWM in resonators has received less attention outside of single pump cases [13].

This Letter explores the FWM phase dependence both in experiments and simulations for integrated resonating cavity systems. The first part focuses on the phase-sensitive nature of nondegenerate FWM and compares experiments with simulations as well as a simplified analytical model. The second part shows, through simulations, the strongly phase-dependent nature of the idlers generated in the modulation-instability (MI) regime, where the nonlinear medium has anomalous dispersion. Our results indicate that control over the relative phase of the initial seed provides a powerful tool that can greatly influence the generated waveform at the system output. Optimization of that phase will therefore be critical for frequency-comb generation in both normal and anomalous dispersion regimes.

In the experiment, a silicon nitride-based microresonator with a Q-value above $7 \times 10^5$ was pumped close to resonance by three phase-locked waves of identical amplitude at 1547.9940, 1548.2260, and 1548.4582 nm with a total pump power of 25 dBm giving an estimated coupled power (to the bus waveguide) of 21 dBm. The pumps were generated using a single tunable external-cavity laser with a specified linewidth of 100 kHz and an intensity modulator driven by a radio-frequency source set to match the free spectral range (FSR) of the resonator cavity at roughly 29 GHz. A pulse shaper (with 10 GHz optical resolution) was then used to tune the phase of the central pump wave. Due to FWM, new lines were then generated inside the microresonator [see Fig. 1(a)]. A schematic of the setup can be seen in Fig. 1(b). The silicon nitride
microresonators used in this experiment were manufactured in a multi-project wafer run using the commercially available dual-layer Triplex technology [14]. The dual-layer technology allows for single-mode operation while maintaining measured linear losses below 0.5 dB/cm. Since silicon nitride, as opposed to pure silicon, lacks two-photon absorption in the optical C-band, it is very promising for high-power applications [15]. Simulations done using a finite-element solver (COMSOL) both verify the single mode property as well as provide an estimated value for the group velocity dispersion. Figure 2 shows the geometry as well as the corresponding simulated mode profile and dispersion curve. It is clearly visible that the waveguide has strong normal dispersion.

To verify the single-mode property in practice, a transmission spectrum was measured by sweeping a single tunable external-cavity laser over a wavelength range covering several FSRs. A part of that scan can be seen in Fig. 3(a). The broadband scan indicates that there exists no coupling to counterpropagating modes nor higher order transverse modes. This ensures that the normal dispersion profile is maintained across the C- and L-band. This is in contrast to recent experiments where interactions between modes produce abrupt changes in the local dispersion of the microresonator [16,17]. Figure 3(b) shows a zoomed-in view of one resonance. When the pump power is increased, owing to thermal effects, the resonance will gradually shift toward the red side when scanning the pump wavelength giving an asymmetric transmission spectrum. This leads to a practical difficulty of pumping close to resonance on the red side, with the detuning parameter $\delta_0 > 0$, but also contributes to thermal stability when pumping on the blue side, where $\delta_0 < 0$ [18]. We will, therefore, even in the simulations, stick to situations where $\delta_0 < 0$. In practice, this also means that it is challenging to achieve phase matching and induce modulation instability in this waveguide platform when pumping with a single CW laser.

Sweeping the phase of the middle pump with the pulse shaper resulted in the generated idler power varying periodically, matching numerical simulations (see Fig. 4). The simulations were done using a standard split-step Fourier nonlinear Schrödinger-equation (NLSE) solver to propagate the light inside the resonator while coupling in the pumps together with quantum noise after every round-trip. In the simulations, the value for the detuning, $\delta_0$, was approximated to be $-0.025$ by matching a measured value for the outcoupled idler power compared to the outcoupled pump power with $\phi_s = 0$ to the corresponding simulated value. The resulting value for $\delta_0$ was then kept for the rest of the simulations and numerical calculations. The results from the measurements show a $\pi$-periodic

![Fig. 1.](image1.png)

**Fig. 1.** (a) Sketch showing the three pumps (black) and the generated idler lines (red) in the foreground with a transmission spectrum showing the relevant resonances in the background. (b) A schematic representation of the setup showing the initial generation block containing the laser, the modulator, and the pulse shaper, as well as the resonator. The two polarization controllers are tuned for optimum operation: the first one for maximum extinction in the intensity modulator and the second one for coupling into the TE mode of the device.

![Fig. 2.](image2.png)

**Fig. 2.** (a) Top view of the layout of the resonator, with a bus waveguide-resonator gap of 1000 nm and a total length of 6031.86 μm. (b) A schematic of the 2D cross-section of the waveguide geometry. (c) Dispersion simulation around 1550 nm for the fundamental TE mode with the inset showing the simulated power distribution of the mode.

![Fig. 3.](image3.png)

**Fig. 3.** (a) Transmission spectrum showing multiple resonances belonging to the single propagating mode. (b) Zoomed-in view showing one resonance swept with a 5-dBm low-powered pump (blue) and a 25-dBm high-powered pump (red). Owing to temperature effects occurring when pumping with high powers, the resonance experiences a shift toward longer wavelengths leading to a triangular shape when sweeping [18].

![Fig. 4.](image4.png)

**Fig. 4.** Outcoupled idler power compared to the outcoupled pump power as a function of the central pump phase, $\phi_s$. The solid black curve shows the calculated ratio using the simplified analytical formula, while the red rings and the black crosses show the simulated and measured ratios, respectively. The offset between the analytical formula and the measured and simulated data is present because of the extra assumptions that are made during its derivation.
power dependence on the central pump phase as well as a variation in the conversion efficiency of 8.0 dB. The split-step simulations confirm the periodicity while having a slightly larger variation at 8.4 dB.

To get a more quantitative understanding of the influence of the resonator parameters in the final idler power, we study the stationary solutions of the Lugiato–Lefever equation [19]. This equation has previously been used in describing the nonlinear dynamics of both fiber-based and microresonator systems [20,21]:

\[
0 = \left( -\frac{\alpha + \theta}{2} - i\delta_0 \right) E_i(t) - i\frac{\beta_2}{2} \frac{\partial^2 E_i(t)}{\partial t^2} + i\gamma L E_i(t) E_\pm(t)^2 + \sqrt{\theta} E_\pm(t),
\]

where the constants \(L, \alpha = a_0 L, \theta, \beta_2\), and \(\gamma\) stand for the resonator length, the total power loss during one round trip, the power coupling coefficient, the group velocity dispersion, and the nonlinearity, respectively. In the experiment, we observed the generation of two new lines on either side of the pumps, so we are interested in stationary solutions, \(E_i(t)\), consisting of five waves:

\[
E_i(t) = \sum_{m=2}^{5} E_m \exp(i m \Omega t),
\]

where \(E_m\) represents the complex amplitude of wave \(m\). The frequency spacing between the waves is denoted with \(\Omega\) and corresponds to the FSR of the resonator. For the input signal \(E_m(t)\), we specify the three-wave pump:

\[
E_m(t) = E_p \exp(-i \Omega t) + i(\phi_i) + \exp(i \Omega t),
\]

where the amplitudes of the three waves are equal, \(|E_p|\), the phases of the two side-pumps are fixed and equal, and the phase of the middle pump wave is shifted by \(\phi_i\) (in practice controlled by the pulse shaper). We can do this without loss of generality, since a linear phase relation between the three lines only corresponds to a time shift of the resulting waveform. By inserting Eqs. (2) and (3) back into Eq. (1) and discarding the terms containing frequencies more than two FSRs away from the central pump, we arrive at a set of five complex valued equations, one for each line. The ones describing the idlers are as follows:

\[
0 = E_{\pm,k} \left( -\frac{\alpha + \theta}{2} + \delta_0 - 2L\gamma P_{\text{out}} + L\gamma |E_{\pm,0}|^2 - 2L\beta_2 \Omega^2 \right) - L\gamma E_p E_k^2 + E_0 E_{\pm,1} + 2E_p E_{\pm,1} E_{\pm,2} - L\gamma E_{\pm,2}^2,
\]

where \(P_{\text{out}} = |E_0|^2 + |E_1|^2 + |E_{\pm,1}|^2 + |E_{\pm,2}|^2\).

To draw some qualitative conclusions about how the idler power depends on various parameters, we will have to make some extra assumptions and simplifications. When pumping a nonlinear material with three pumps, there are several FWM processes acting simultaneously. These will correspond to different terms in the equation above. Some of the processes have already been ignored when avoiding the frequency terms far away from the pumps, where the power levels are low [22]. We will from here on also ignore the two symmetric FWM processes that couple the two idlers together: the degenerate one transferring power from the middle pump to the two idlers and the nondegenerate one transferring power from the two side pumps to the two idlers. These assumptions, removing the terms containing \(E_{\pm,2}\), allow us to simplify Eq. (4) retaining the main features of the system behavior, and permit recasting it into the following closed-form expression:

\[
|E_{\pm,2}|^2 = \frac{|E_0|^6 L^2 \gamma^2 (5 + 4 \cos(2\phi))}{\alpha^2 + (6|E_0|^2 \gamma + 2\beta_2 \Omega^2)^2},
\]

where \(\alpha_0\) corresponds to the propagation losses per length, while the rest of the parameters are defined as before. The significant difference between the ring and the waveguide case is that for the waveguide, it is the pump power, whereas for the resonator system it is the in-cavity power that appears. To draw some conclusions, we can take an additional step: assume that, for the resonating system, \(\delta_0 = 0\), the dispersion is optimal, and that we have critical coupling \((\theta = \alpha)\). The in-cavity idler power will then be proportional to \(1/\alpha^2\), and the outcoupled one will be proportional to \(1/\alpha^3\). This is in comparison with the straight waveguide case, where the output idler power will remain proportional to \(1/\alpha^2\). In the low-loss regime, pumping a resonator instead of a waveguide will thus greatly enhance the output idler power.

The previous analysis dealt with resonant wavelength conversion via FWM. However, the thermal locking dynamics together with the normal dispersion of the waveguide prevented the formation of MI and hence parametric gain. In the anomalous dispersion regime, when the power levels are high enough,
combs can be generated using a single continuous wave pump (see analysis in [20,21,24]). To investigate the phase dependence in such a case, we turn to the split-step NLSE simulations. The analytical model developed in the previous paragraph will not be valid in general as we discarded the waves located far away from the pump in the frequency domain. When using several pumps to seed the comb, lower power levels can already cause MI growth [5,7]. In these cases, however, the relative pump phases also have great effect on the final power in the MI sidebands. Figure 5 shows the results from a simulation demonstrating such a case. The simulations were done using three equally powered pumps, placed slightly off resonance on the blue, thermally stable, side. The ring parameters were chosen to be realistic and are specified in the figure caption. Setting the pump phase difference from $\phi_1 = 0$ to $\phi_1 = \pi/2$ gives significantly different results: in the first case, we see MI sidebands of significant amplitudes, whereas in the second case, there is no new frequency generation at all, giving a phase-dependent relative conversion efficiency for the generated idlers of above 40 dB. In the time-domain picture, when the MI gain is present, an interference pattern corresponding to the distance between the pump laser and the MI generated sidebands is also visible. These results indicate that the relative phase between the input lines drastically alter the gain profile of the MI. Further investigations are needed to find the limits and the full extent of this dependence.

In summary, we have demonstrated triply resonant four-wave mixing in a silicon nitride microresonator and verified the periodic phase dependence of the results using both split-step NLSE simulations and a simplified analytical expression derived from the Lugiato–Lefever model. We have also demonstrated simulations showing a strong phase dependence of modulation instability gain giving rise to a conversion efficiency difference of above 40 dB in resonators with anomalous dispersion. The control and optimization of the relative phases of a multiple-pump seed thus provides a powerful tool in controlling the final waveform, with implications for applications both in the normal and the anomalous dispersion regime.

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